Solutions: Middleton Invitational 2-18-2006 Geometry Team SOLUTIONS

1. C A D B from definitions

2. Euler's Formula for solids: vertices + faces = edges + 2
A (number of vertices) + 12 = 24 + 2 A = 14
For diagonals in an n-gon, use n (n - 3)/2 For a dodecagon, n = 12. B = 12(12 - 3)/2 = 54
C = 360/9 = 40 D = 1 + 3 + 6 + 10 + 15 + 21 = 56
A + B + C + D = 14 + 54 + 40 + 56 = 164

3. Volume of a cylinder = $\pi r^2 h$, where h is the height. $180\pi = \pi\left(\frac{12}{2}\right)^2 h$. A = 5

Surface area of a cone = $\pi r^2 + \pi r l$ where r = radius of the base of the cone and l is the slant height. $\pi r^2 + \pi r(9) = 90\pi$ Divide pi out of both sides and solve the quadratic. $r^2 + 9r - 90 = 0$. Factors are (r + 15) and (r - 6). For a positive r, r = 6. B = 6

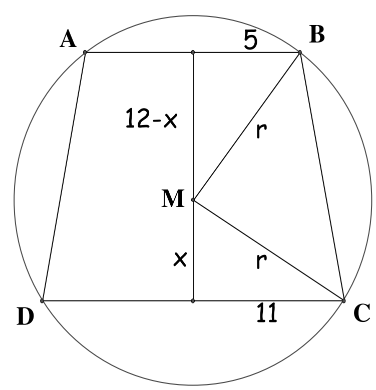
Volume of a square pyramid = $\frac{1}{3}s^2 h$ where s = side length of the square base and h is the height. $400 = \frac{1}{3}s^2(12)$. $100 = s^2$ s = 10 C = 10

Surface area = 2(lw) + 2(lh) + 2(wh). 472 = 2(12)w + 2(12)(7) + 2(w)(7). w = 8 D = 8

A + B + C + D = 5 + 6 + 10 + 8 = 29

4. First find the height of triangle ACE. $h = \frac{10 \bullet 6}{10+6}$ The area of the triangle is $\frac{1}{2} \bullet 12 \bullet h = 6 \bullet \frac{15}{4} = \frac{45}{2}$ or $27\frac{1}{2}$

5. To find the radius begin by constructing the height through the center of the circle and solving two right triangle problems to find x. $r^2 = 5^2 + (12 - x)^2$ and $r^2 = 11^2 + x^2$ so that $25 + 144 - 24x + x^2 = 121 + x^2$ and x = 2. $r^2 = 121 + 4 = 125$
Area = $\pi r^2 = 125\pi$



6. Combine like terms. Arrange in circle format. Complete the square.

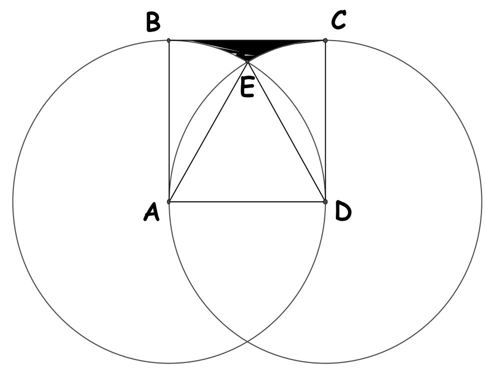
$x^2 - 10x + \_\_\_ + y^2 - 4y + \_\_\_ = -13$ $x^2 - 8x + \_\_\_ + y^2 + 6y + \_\_\_ = -3 + 14$

$x^2 - 10x + \underline{25} + y^2 - 4y + \underline{4} = -13 + 25 + 4$ $x^2 - 8x + \underline{16} + y^2 + 6y + \underline{9} = 11 + 16 + 9$

$(x-5)^2 + (y-2)^2 = 16$ center (5, 2) radius 4 $(x-4)^2 + (y+3)^2 = 36$ center (4, -3) radius 6

A + B + C + D + E + F = 5 + 2 + 4 + 4 + (-3) + 6 = 18

7. Label the point of intersection of the two circles inside of ABCD as E. AED is equilateral with sides of 10. Area equals $\frac{10^2\sqrt{3}}{4} = 25\sqrt{3}$. The area of each of the sectors ABE and DCE is $\frac{30}{360} \bullet \pi \bullet 10^2 = \frac{25\pi}{3}$. The shaded area is the area of ABCD minus the area of the triangle and both of the sectors.
Area = $100 - 25\sqrt{3} - \frac{50\pi}{3}$

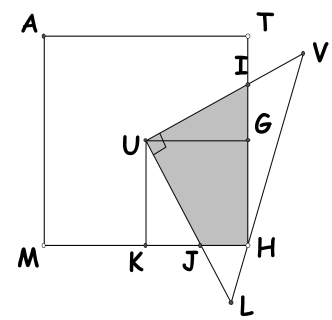


8.

A	True	+5	
B	False	-13	
C	True	+23	
D	False	-37	
E	True	+41	
F	True	+59	Total is 78

9. The x and y axis make this a right triangle with one leg equal to the y intercept of 9. To find the x intercept, solve the equation for y = 0. x = 6 Area = .5(9)(6) = 27

10. $\angle GUI \cong \angle KUJ$ because both are 90 - m$\angle GUJ$. Also, both are right triangles with a leg of 5. Their areas are equal. The shaded area is simply the area of square KUGH. $5^2 = 25$.

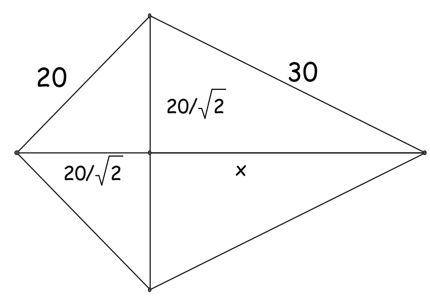


11.

Day	1	2	3	4	...	n
Total Nuts	5	18	35	56		
Factors	1•5	3•6	5•7	7•8	...	(2n-1)(n+4)

For n = 30 $(2 \bullet 30 - 1)(30 + 4) = 2006$

12. The 90° angle has to be between the 20 cm sides because $30\sqrt{2} > 2 \bullet 20$. One diagonal is $20\sqrt{2}$. To find the other, use the Pythagorean Theorem to find $x = 10\sqrt{7}$. The area of the kite is half the product of the diagonals.
$\frac{1}{2}(20\sqrt{2})(10\sqrt{2} + 10\sqrt{7}) =$ $200 + 100\sqrt{14})$



13. The centroid is located at the mean of the vertex coordinates.
$\left(\frac{-13+22-15}{3}, \frac{7-14+13}{3}\right)$ which is (-2, 2)

14. For $\Delta ABC \sim \Delta MNP$, a scale factor of 3 times 17cm will produce a side of 51 and integer side lengths for all three sides in the not shown triangle. The largest possible side is 3(24) = 72 = K. For $\Delta DEF \sim \Delta XYZ$, a scale factor of 5 times the 17 cm side will produce a side length of 85 and integer side lengths in the not shown triangle. Also, ΔDEF is a right triangle. The largest possible altitude is the longest leg. 5(15) = 75 = H.
K + H = 72 + 75 = 147

15. ΔABG is a right triangle with legs of 3 and 4. BG is 5. ΔABG is similar to ΔDHG. To find DH, solve the proportion $\frac{3}{4} = \frac{DH}{1}$ to find DH = $\frac{3}{4}$ so that CH is $\frac{9}{4}$ and the area of ΔBCH is $\frac{1}{2} \cdot \frac{3}{1} \cdot \frac{9}{4} = \frac{27}{8} = 3.375$