Sponsor's Copy Middleton Invitational Calculus 2-18-2006

1. Let A be the set of value(s) of c which satisfies the conclusion of the Mean Value Theorem (for derivatives) for $f(x) = x^3 - 2x^2 + x$ on the interval [0, 1].

   Let B be the set of value(s) of $f''(x)$ when $f'(x) = 1$ for $f(x) = x^3 - 2x^2 + x$, where $x > 0$.

   Give the sum of all members of $A \cup B$.

2. Let $f(x) = -2x^3 + 2\sqrt{x} + 8x + 3$

   Let A be the slope of the line normal to f at the point (1, 11).

   Let B be the slope of the line tangent to f at $x = 4$.

   Let C be the slope of the inverse $f^{-1}(x)$ at the point on $f^{-1}(x)$ where $x = 11$.

   Give the value of $A \bullet B \bullet C$.

3. Let A be the value of $\lim_{x \to 9} \dfrac{x-9}{x-\sqrt{3}}$. If the limit does not exist, let $A = 10$.

   Let B be the value of $\lim_{h \to 0} \dfrac{(3+h)^3 - 2(3+h)^2 - (27) + 2(9)}{h}$. If the limit does not exist, then let $B = 20$.

   Let C and D be the values for which $f(x) = \begin{cases} Cx^2 + Dx + 1 & for \quad x \le 2 \\ 13 - Cx & for \quad x > 2 \end{cases}$ is both continuous and differentiable.

   Give the value of $A + B + C + D$.

4. For $y = \sec^4(2x^2)$ at let $\dfrac{dy}{dx} = Ax(\sec(2x^2))^P \tan(2x^2)$

   Let B be the value of x where the graph of g has a relative minimum, given that $g'(x) = (x-2)^2(x+3)(4-x)$.

   Let C be the value of $\dfrac{d^2y}{dx^2}$ for $t = 1$ if $y = 2t$ and $x = t^3$.

   Give the value of $\dfrac{9 \bullet A \bullet B \bullet C \bullet P}{16}$.

5. For $f(x) = x^2 - \dfrac{1}{x}$, let S be the set of integers for which the graph of f is concave up. Do not include inflection points.

   Let T be the set of integers for which the graph of f is increasing.

   Let Q be the values of $T \cap S$ which are in the domain of f and which satisfy the inequality $|x| < 5$.

   Give the members, in order, of Q.

6. $f(x) = \left| \, |x-6| - 6 \, \right|$

Let A be the value of $f\,'(-1)$.

Let B be the set of x value(s) for which there is a critical point on the graph of f.

Let C be the maximum value of f over the interval [0, 6].

Give the sum of A, C and all members of set B.

7. A single term for the term a_n sequence $-5,\ 9,\ -5,\ 9,\ -5,\ 9,\ \ldots$

given that for $n=1$ the first term is a_1, is $A(-1)^n + B$.

Let C the value of $\dfrac{d^2y}{dx^2}$ at the point $(6,-8)$ on the circle with equation $x^2 + y^2 = 100$.

Give the product $64 \bullet A \bullet B \bullet C$.

8. Consider the velocity of a particle, moving along the x-axis $v(t) = t^2 - 6t + 5$ in units per minute for $t \geq 0$ minutes.

Let (S, V) be the second complete interval of t when the particle is slowing down.

Let (T, W) be the complete interval of t when the particle is moving to the left.

Let value U units per minute^2 be the maximum acceleration of the particle over the time interval [0, 4].

Give the sum $S + V + T + W + U$.

9. Using differentials to approximate $\sqrt{96}$ gives value $\dfrac{A}{5}$.

Using differentials to approximate $\sqrt[3]{28}$ gives value $\dfrac{B}{27}$.

Using differentials to approximate $\dfrac{1}{\sqrt{9.2}}$ gives a value of $\dfrac{1}{3} - \dfrac{1}{C}$.

Give the value of $A + B + C$.

10. The graph of a continuous function f has a horizontal normal line at the point (1, 4) on the curve. The equation of the tangent line at that point is $Ax + By = 8$.

If $G(x) = \displaystyle\int_1^{2x} \frac{1}{1+t^2}\,dt$ then let C the value of $G'(2)$.

Give the value of $A \bullet C + B$.

11. The volume of a cube is increasing at the rate of 20 cubic cm per second. When the edge is 10 cm…

its surface area is increasing at S square cm per second,

its diagonal is increasing at D cm per second,

its shadow is a parallelogram with base and height equal to the length of the cube's edge. The rate that the area of the parallelogram is changing is P sq. cm.

Give the value of $S \bullet D \bullet P$.

12. An isosceles triangle has two sides with length 8 and included angle θ. If the legs stay constant and θ is increasing at $\frac{\pi}{180}$ radians per minute, then

let A be the rate of change of the area of the triangle when θ is $\frac{\pi}{3}$,

let B be the distance from the vertex of the triangle to the base, when θ is $\frac{\pi}{3}$.

Find the value of $\frac{A \bullet B}{\pi\sqrt{3}}$.

13. $f(x) = \frac{3x\sqrt{x+2}}{x-1}$ and $f'(2) = A$

$g(x) = 4\sin(3-x)\cos(3-x)$ and $g'(2) = B\cos C$

$h(x) = e \bullet e^{1-x}$ and $h'(2) = D$

Give the value of $(A \bullet B) + C + D$.

14. $f(x) = 6x^2$ and $g(x) = 5x - 1$ intersects at the points (A, B) and (C, D), for $A < C$.

Let E be the value of $f'(A)$ and let F be the value of $g'(C)$.

Give the value of $\frac{E \bullet F}{A \bullet C}$.

15.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	8	2	-1	4
2	4	5	-2	5
-1	A	B	C	D

f and g are continuous and twice-differentiable functions, both defined over all reals.

$h(x) = f(g(x))$.

f is an even function, and g is an odd function.

Let P be the value of $h'(1)$.

Give the sum A+B+C+D+P.