

NO CALCULATOR!

Middleton Invitational 2/18/2006

The abbreviation NOTA denotes  
None of These Answers.

1. How many zeros are at the end of the expansion of  $126!$  ?

A. 25      B. 30  
C. 31      D. 32  
E. NOTA

2. Let  $x = \sqrt{132 + \sqrt{132 + \sqrt{132 + \sqrt{\dots}}}}$  and  
let  $y = \sqrt{132 - \sqrt{132 - \sqrt{132 - \sqrt{\dots}}}}$  then  
which is the value of  $x + y$ .

A. 0      B. 23  
C. 24      D. 26  
E. NOTA

3. Evaluate  $\sin(\cos^{-1}\frac{1}{9})$ .

A.  $\frac{8}{9}$       B.  $\frac{9}{4\sqrt{5}}$   
C.  $\frac{4\sqrt{5}}{9}$       D.  $4\sqrt{5}$   
E. NOTA

4. What is the amplitude times the period of the graph of  $y = 13\sin x \cos x$  ?

A.  $26\pi$       B.  $13\pi$   
C.  $\frac{13}{2}\pi$       D.  $\frac{\sqrt{13}}{2}\pi$   
E. NOTA

5. Simplify  $\frac{\cot \theta}{\cos \theta - 2\sin^2 \theta \cos \theta}$   
completely for  $0 < \theta < \frac{\pi}{4}$  ?

A.  $\frac{1}{\sin(3\theta)}$   
B.  $\frac{1}{\sin \theta \cos(2\theta)}$   
C.  $\frac{1}{\cos \theta \sin \theta}$   
D.  $\frac{\sin \theta}{\cos^2 \theta \cos(2\theta)}$   
E. NOTA

6. How many distinct triangles  $ABC$  can be made with  $m\angle A = 30^\circ$ ,  $BC = 2$ ,  $AC = 3$ ?

A. 2      B. 1  
C. 0      D. infinitely many  
E. NOTA

7. What is the amplitude of the graph of  $y = -2\sin(x) + 7\cos(x)$  ?

A. 9      B. 11  
C.  $2\sqrt{7}$       D.  $\sqrt{53}$   
E. NOTA

8. What value of  $k$  will make the vectors  $\langle 2, -7, k \rangle$  and  $\langle 3, 2, 6 \rangle$  orthogonal?

A. -2      B.  $-\frac{4}{3}$   
C.  $\frac{4}{3}$       D.  $\frac{7}{6}$       E. NOTA

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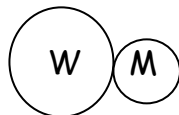
9. In triangle MHS, with MH=3, MS=4 and HS=2, determine the area of MHS.

- A. 12      B.  $\frac{3\sqrt{15}}{4}$   
 C.  $\frac{3\sqrt{15}}{2}$       D.  $\frac{5\sqrt{5}}{4}$   
 E. NOTA

10. If the base-ten number 60 is written in base two, what is the sum of the digits?

- A. 4      B. 5  
 C. 6      D. 7  
 E. NOTA

11. If circle W has a radius of 6 times the radius of circle M, and the circles are externally tangent to each other, then how many revolutions will M make if W makes one revolution?



- A. 6      B. 5  
 C. 4      D. 3  
 E. NOTA

12. Evaluate  $i^{2006}$  for  $i = \sqrt{-1}$ .

- A.  $i$       B. 1  
 C.  $-i$       D.  $-1$   
 E. NOTA

13. If  $\frac{|3i+4|}{1-i} = a+bi$  then  $a+b =$

- A. 5      B. 5.5  
 C. 7      D. 7.5  
 E. NOTA

14. Find the sum of the rational roots of the equation  $x^3 - x^2 - 3x - 1 = 0$ .

- A. 2      B. 1  
 C. -1      D. -2  
 E. NOTA

15. The graph of the polar equation

$$r = \frac{3}{3 - 4\cos\theta}$$

is a

- A. ellipse      B. hyperbola  
 C. lemniscate      D. limaçon  
 E. NOTA

16. If  $x$  and  $y$  are real numbers then what is the domain of  $y = \sqrt{3-x^2}$  ?

- A.  $|x| \leq \sqrt{3}$       B.  $|x| < \sqrt{3}$   
 C.  $|x| \geq \sqrt{3}$       D. all reals  
 E. NOTA

17. Find the sum of the positive integral factors of 10.

- A. 7      B. 8  
 C. 10      D. 18  
 E. NOTA

18. Evaluate  $(\log_2 9) \cdot (\log_3 \sqrt{8})$ .

- A. 1      B. 1.5  
 C. 2      D. 6  
 E. NOTA

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19. If  $J + U = 4$  and  $J^2 + U^2 = 20$  then give the value of  $2 \cdot J \cdot U$ .

- A. 16      B. 12  
C. -4      D.  $2i$   
E. NOTA

20. Express  $\sqrt{396}$  in simplest radical form,  $a\sqrt{b}$  and then give  $a + b$ .

- A. 11      B. 17  
C. 19      D. 20  
E. NOTA

21.  $4(\cos 60^\circ)^{-\frac{3}{2}} =$

- A.  $8\sqrt{2}$       B.  $\sqrt[3]{3}$   
C.  $\frac{\sqrt{2}}{2}$       D.  $4\sqrt{2}$   
E. NOTA

22. Jorge walked 12 miles uphill in 30 minutes, and then 12 miles downhill at a rate of 4 miles per hour. What was Jorge's average speed in miles per hour for the entire trip?

- A. 5 mph      B. 14 mph  
C.  $6\frac{6}{7}$  mph      D.  $4\frac{4}{5}$  mph  
E. NOTA

23. Will walked 8 miles, downhill, at 4 miles per hour, then completed the 24-mile-total downhill trip by falling, laying still for 10 minutes, then rolling the rest of the trip. If his average rate for the downhill journey (walking, lying and rolling) was 6 miles per hour, find the rate in miles per hour that Will rolled.

- A.  $13\frac{5}{6}$       B.  $8\frac{8}{11}$   
C.  $2\frac{1}{3}$       D.  $2\frac{1}{6}$   
E. NOTA

24. Let  $A$  be the smallest prime greater than 49, and  $B$  be the greatest prime less than 100, and  $C$  be the smallest whole number, then give the value of  $A+B+C$ .

- A. 149      B. 150  
C. 151      D. 152  
E. NOTA

25. The graph of  $y = \frac{x^2 + 1}{x - 1}$  approaches the line  $y = mx + b$  as  $x$  approaches positive infinity. What is the value of  $3m + b$  ?

- A. 0      B. 4  
C. 5      D. 6  
E. NOTA

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26. The graphs of  $r = 4\cos\theta$  and  $r = 2$  meet at the points R and S. Give the length of the minor arc  $\widehat{RS}$  on the graph of  $r = 4\cos\theta$ .

- A.  $\frac{4\pi}{3}$       B.  $\frac{2\pi}{3}$   
 C.  $\frac{\pi}{6}$       D.  $\frac{\pi}{8}$   
 E. NOTA

27. The three third-roots of  $(8i)$  are  $ai$ ,  $b+ci$  and  $b-ci$ . Give the value of  $a^2+c^2$ .

- A. 6      B. 5  
 C. 4      D. 3  
 E. NOTA

28. A regular octagon with side length 20 is inscribed in a circle. Let  $x = \sin 22.5^\circ$ ,  $y = \cos 22.5^\circ$  and  $z = \tan 22.5^\circ$ . Which is an expression for the area outside of the octagon and inside of the circle?

- A.  $\frac{50\pi x^2 - 800}{y}$       B.  $\frac{100\pi - 800xy}{x^2}$   
 C.  $\frac{100\pi x - 400y}{z}$       D.  $100\pi x^2 - 800z$   
 E. NOTA

29. Which is an expression for  $\cos(\text{Arc cos}(x) - \text{Arc sin}(x))$  for  $0 < x < 1$ ?

- A.  $\frac{1}{x^2 - x^4}$       B.  $x^2 - x^4$   
 C.  $2x\sqrt{1-x^2}$       D.  $\frac{\sqrt{1-x^2}}{2x}$   
 E. NOTA

30. If  $\sqrt{2x+1} - \sqrt{x-3} = 4$  for real value of  $x$ , then what is the value of  $\sqrt{100-x}$ ?

- A. 16      B. 8  
 C. 6      D. 4  
 E. NOTA