

NO CALCULATOR!

Middleton Invitational 2/18/2006

Solutions:

- Divide by 5, by 25 and by 125. Add the results and you get 31. **C**.
- Short cut: $132 = 12(11)$ so the expression with the "addition" is the larger and the "subtraction" is the smaller. $x=12$ and $y=11$. Sum 23. **B**.
- Draw a right triangle in quadrant I and the $x=1$, the hypotenuse is 9 so $y=\sqrt{80}$. The sin is then $\frac{4\sqrt{5}}{9}$ which is **C**.
- The expression is equal to $\frac{13}{2}\sin(2x)$ by the double-angle sine property. So the amp=6.5 and the period is 2π divided by the coefficient 2, to give the answer is $\frac{13}{2}\cdot\pi = \underline{\mathbf{C}}$.
- $\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\cos\theta(\cos 2\theta)}$ by factoring out a cosine from the bottom right and using the cosine double-angle rule. simplify to choice **B**.
- AC times sinA gives 1.5, and since 2 (side opposite A) is more than 1.5 and less than 3, there are two solutions. **A**.
- $\sqrt{2^2+7^2} = \underline{\mathbf{D}}$.
- The dot product must be 0:
 $6-14+6k=0$ so $k=4/3$. Answer **C**.
- Using Heron's formula: $s=9/2$
$$\sqrt{\frac{9}{2}\left(\frac{9}{2}-\frac{6}{2}\right)\left(\frac{9}{2}-\frac{8}{2}\right)\left(\frac{9}{2}-\frac{4}{2}\right)} = \sqrt{\frac{9(3)(5)}{16}}$$
$$= \frac{3\sqrt{15}}{4} = \underline{\mathbf{B}}$$
- In base 2, we get 111100 for a digit sum of 4. Choice **A**.
- The circumferences will be in the same ratio as the radii. Answer **A**.
- $i^{2006} = i^2 = -1$. Choice **D**.
- The numerator is 5, so $\frac{5}{1-i} = \frac{5(1+i)}{2}$ and $a+b = 5/2 + 5/2 = 5$. Choice **A**.
- 1 is a root. Divide by this and you get that the other two roots are irrational. So the sum of the rational roots is -1. **C**.
- Change to rectangular form, or use the rule of coefficients. If you do the former, you get $3r - 4r\cos\theta = 3$ and $3\sqrt{x^2+y^2} - 4x = 3$ then $3\sqrt{x^2+y^2} = 3+4x$ which we square to get $9(x^2+y^2) = 14x^2 + 24x + 9$ which is a hyperbola. Choice **B**.
- $|x| \leq \sqrt{3}$ since $3-x^2 \geq 0$. Choice **A**.
- $1+2+5+10 = \underline{\mathbf{D}}$.
- $\frac{2\log 3}{\log 2} \cdot \frac{1}{2} \cdot 3 \cdot \log 2 = 3$. Choice **E**.
- Square the first equation to get $J^2 + 2JU + U^2 = 16$. Substitute the 2nd equation and subtract to get $2JU = -4$. Choice **C**.
- $6\sqrt{11}$ gives 6+11 is 17. Choice **B**.
- $4(\cos 60^\circ)^{-\frac{3}{2}} = 4\left(\frac{1}{2}\right)^{-\frac{3}{2}} = 4(2)^{\frac{3}{2}} = 4\sqrt{8}$
which is $8\sqrt{2}$ or choice **A**.
- total distance / total time = $24/(.5+3) = 24/3.5 = 240/35 = 48/7 = \underline{\mathbf{C}}$.
- distance/time = $24/(2+1/6+x) = 6$ solves to time rolling is 11/6. So rate is 16 miles rolling divided by time 11/6 gives answer **B**.
- $53+97+0 = 150$. **B**.
- Divide to get $y = x+1 + \frac{2}{x-1}$ and the slant asymptote is $y=x+1$ for $3m+b=4$. **B**.

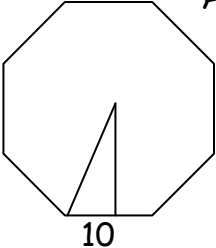
Alpha Open Test

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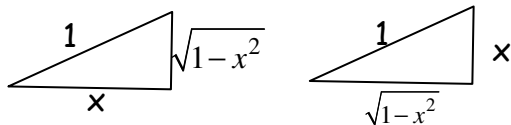
26. Set the equations equal and we get $\cos \theta = \frac{1}{2}$ and they meet when $\theta = \frac{\pi}{3}$ and $\theta = -\frac{\pi}{3}$ for an angle of $\frac{2\pi}{3}$. Both graphs are circles with a radius of 2, so $\frac{2\pi}{3}$ is $\frac{1}{3}$ of the circumference which gives $\frac{4\pi}{3}$. Choice **A**.

27. Using DeMoivre's theorem, to get $2\text{cis}30$, $2\text{cis}150$ and $2\text{cis}270$ in degrees, which gives $-2i$, $\pm\sqrt{3}+i$ and $a^2+c^2=5$ which is choice **B**.

28.  Area is $\frac{1}{2}$ times apothem times perimeter or $\frac{1}{2}$ times $10/\tan 22.5$ times 160. This gives area of the

octagon is $800/z$. The circle's radius is $10/\sin 22.5$ which is $10/x$. So the area of the requested portion is $100\pi/x^2 - 800/z$. But if we change the last part to $800y/x$ and get a common denominator, we get choice **B**.

29. $\cos(\text{Arc cos}(x) - \text{Arc sin}(x))$
 $= \cos(\text{Arc cos } x) \cos(\text{Arc sin } x) + \sin(\text{Arc cos } x) \sin(\text{Arc sin } x)$



$$= x(\sqrt{1-x^2}) + \sqrt{1-x^2}(x) = 2x\sqrt{1-x^2} = \mathbf{C}$$

30. Square to get $\sqrt{2x+1} = 4 + \sqrt{x-3}$
 $2x+1 = 16+x-3+8\sqrt{x-3}$ or
 $x-12 = 8\sqrt{x-3}$
 $x^2 - 24x + 144 = 64(x-3)$ or
 $x^2 - 88x + 336 = 0$
 $(x-4)(x-84) = 0$ (factor 336 to 3, 4, 4, 7 and to get a large 88 we use $3(4)(7)$ and 4. The answer $x=4$ does not give a true equality, so we discard it. The answer $x=84$ works, and so the square root of $100-x$ is 4. Answer **D**.