Middleton Invitational 2006 (No Calculator Permitted)

Algebra II Team Round Solutions

1. **A** = $\sqrt{4^2+4^2} = 4\sqrt{2}$
   B = f (2-7) = f (5) = 2(2^2) - 31(2) + 104 = 50
   C! = (23-1)**!** = 22**!** = 22

$$\frac{2|C - B|}{A} = \frac{28*2}{4\sqrt{2}} = 7\sqrt{2}$$

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2. **A** = $(39 - 41)^{10} = 2^{10}$ *or* 1024
   B = $\frac{-17545 - 3829i}{17545 + 3829i} = -1$
   C
   $= \frac{(x-9)^2}{1024} + \frac{(y+18)^2}{1024} = 1$, cirle, $r^2 = 1024$, $area = 1024\pi$
   D = $\frac{55-73}{62-68} = \frac{-18}{-6} = 3$

$$\left(\frac{BCD}{A\pi}\right)^B = \left(\frac{-1*1024\pi*3}{1024\pi}\right)^{-1} = -3^{-1}\ or\ -\frac{1}{3}$$

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3. **A** = $x = 3+\frac{2}{1+\frac{5}{x}}, x = 3+\frac{2}{\frac{5+x}{x}}, x = 3+\frac{2x}{5+x}$
   $(x-3)(x+5) = 2x, x^2 - 15 = 0, x = \sqrt{15}$

$$\begin{bmatrix} 3 & E \\ -7 & P \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ 2P & E \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ -35 & C \end{bmatrix}$$

$E - 1 = 6, E = 7$

$-7 + 2P = -35, P = -14$

$P + E = 7 - 14 = -7 = C$

$$-C + A = 7 + \sqrt{15}$$

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4. **A** = $\frac{60}{360} * \pi r^2 = 24\pi, r^2 = 24*6, r^2 = 144$
   $SA = 4\pi r^2 = 4(144)(\pi) = 576\pi$

   B = $\frac{1}{3}(6)(\pi)(144+36+12*6) = 2\pi(252) = 504\pi$
   C
   $= \sqrt{3}\sqrt{\frac{1}{8}+\frac{1}{16}} - 4 = \sqrt{3}*\frac{\sqrt{3}}{4} - 4 = \frac{3}{4} - \frac{16}{5} = -\frac{13}{4}.$
   $13 - 4 = 9$
   D = $x^{24} - 8^4 = 0, x^{24} = 2^{12}, |x| = \sqrt{2}$

$$D\left(\frac{A - B}{C\pi}\right) = \sqrt{2}*\left(\frac{72\pi}{9\pi}\right) = 8\sqrt{2}$$

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5. **A =** $\sum_{0}^{6} x^2 = \frac{n(2n+1)(n+1)}{6} = \frac{6(13)(7)}{6} = 91.$

$9+1=10$

B = 2

$$\begin{array}{rl} & 3x + 6y - 4z = 14 \\ \mathbf{C} = \; + & -7x + 2y + 8z = 16 \\ & -x - 3y - 9z = -10 \\ \hline & -5x + 5y - 5z = 20 \end{array} \quad , \; x - y + z = -4$$

D = the value of x in, $(\log_x 10^4) + 3\log_x 10^2 = 10$

$\log_x 10^{10} = 10, x = 10$

$(C)\left(B - \frac{A}{D}\right)$

$(-4)\left(2 - \frac{10}{10}\right)$

$= -4$

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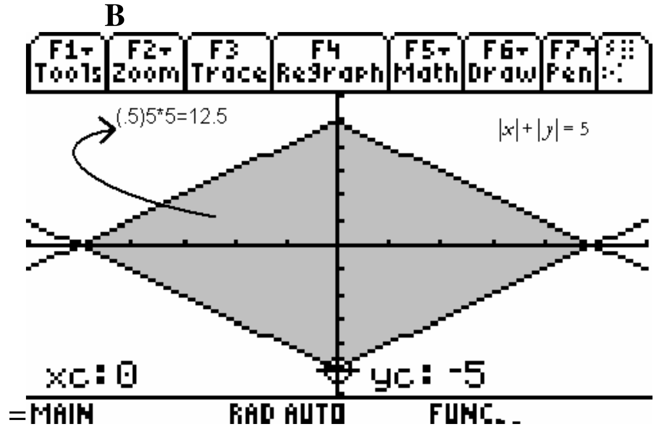
6. **A** = the number of zeros in 25**!** is given by the sum of the remainders when 25 is divided by 5. **A** = 6.

B



12.5*4 = 50

C = $3r^3 = 375, r^3 = 125, r = 5$

D = $2x^3 - 10x^2 + 12 = 0, \; \frac{-b}{a} = \frac{-(-10)}{2} = 5$

$A^{\left(\frac{B}{CD}\right)}$

$= 6^{\frac{50}{25}}$

$= 6^2 \; or \; 36$

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7. **A** = 729 @ 23 = $729^{\frac{1}{23-17}} = \left(3^6\right)^{\frac{1}{6}} = 3.$

B =
$\frac{a}{180-a} = \frac{3}{7}, 7a + 3a = 180 * 3$

$a = 18 * 3 = 54$

$90 - 54 = 36$

$\frac{54}{36} = \frac{3}{2}$

$A^{-1}B + \mathbf{C}$

$= \frac{1}{3} * \frac{3}{2}$

$= \frac{1}{2}$

C = $x^2 - 6x - 8y = -25$. Axis of symmetry, x =3.
Directrix, y = 0.
3*0 = 0

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8. **A** = 2
B = $-i\sqrt{2}$
C = $i\sqrt{2}$
D = 2
E = 0

$$\frac{B}{C} + AD - E = \frac{i\sqrt{2}}{-i\sqrt{2}} + 4 - 0 = -1 + 4 = 3$$

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9. **A** = the median in a right triangle is half the length of the hypotenuse. Therefore, the hypotenuse is 50, so the triangle must have sides 2(7-24-25) = 14-48-50. The area would be $A = \frac{1}{2}(14)(48) = (14)(24) = 336$

B = $x + \frac{6}{x} = 5$, $x^2 - 5x + 6 = 0$, x = -3 or x = -2.
It turns out that both answer yield the same result.13.

A+B
=336+13
=349

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10. **A** = $3(1) - 5 = -2$
$1 + (-2)^2 = 5$
$5^3 - 1 = 124$

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{225} = 1$$

$a = 15$ b=3 c=$\sqrt{221}$

B = $\frac{c}{a} = \frac{\sqrt{221}}{15}$

Cπ = $ab\pi = 15(2)\pi = 30\pi$. 30

$$BC\sqrt{A + 101} = 30\left(\frac{\sqrt{221}}{15}\right)\left(\sqrt{124+101}\right) = 30\sqrt{221}$$

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11. A = $\sum_{x=1}^{100} x = \frac{100(101)}{2} = 5050$

B = $\sum_{x=0}^{\infty} 16\left(\frac{1}{2}\right)^x = \frac{16}{1 - \frac{1}{2}} = 16 * 2 = 32$

C =Midpoint, M: (4,1)

$d = \sqrt{(1+3)^2 + (4-1)^2} = \sqrt{16+9} = 5$

$$\frac{AC^{-1}}{B} = \frac{5050}{5 * 32} = \frac{1010}{32} = \frac{505}{16}$$

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$$\sqrt{C}\left(\frac{A+C}{B-D}\right)^{D}$$
$$=\sqrt{3}(1)$$
$$=\sqrt{3}$$

12. A = -1
B = 50
C = 3
D = $2x^2 - y^2 = 12$, $xy = 7$, $y = \frac{7}{x}$. From here we $2x^4 - 7x^2 - 12 = 0$. Let $p = x^2$.
can determine that the roots of this equation are opposites, which will yield a sum of zero.

Find the absolute value of the difference between the true and false statements.
$$|(2*2)-(3*(-3))|$$
$$=|4+9|$$
$$=13$$

13. True=2. False= -3.

0! Is equal to one. [2]

The quotient of the eccentricity of a hyperbola and the eccentricity of a circle is negative. [-3]

π is a real number. [2]

$\sqrt{\frac{4}{9}}$ is an irrational number. [-3]

For any real number p, $p \bullet \frac{1}{p} = 1$ by the identity property of equality. [-3]

= A+B+C+D+E
= 16+6+3+2+1
= 28

14. $\frac{\log x}{\log 2}+\frac{\log x}{2\log 2}=1$ and getting a common denominator gives $\frac{2\log x}{2\log 2}+\frac{\log x}{2\log 2}=1$
and so $\log(x^3)=\log 4$ and $x^3=4,\ x^6=16$. So A=16.
For part II, multiply by the conjugate to get
$$\frac{\sqrt{6}+\sqrt{3}+\sqrt{2}+\sqrt{1}}{2}.$$

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6A+D+E
= 13+7
= 20

15.Get a common denominator to get $4x+1=A(x+3)+B(x-3)$ and set the coefficient of the x's equal to get A+B=4, and the constants are equal to get 3A-3B=1. We get 6A=13.
On part II, square both sides to get D+E=7 and DE=10 so D and E are 5 and 2 in any order.