# SOLUTIONS

```
       0  r2
 7)    2  r6
7)    20
```

1. **(A)** The slope is $\frac{1-(-4)}{3-2}=5$. Manipulating $y+4=5(x-2)$ gives the answer.
2. **(B)** $(13)_5=3(5^0)+1(5^1)=8$. $(13)_9=3(9^0)+1(9^1)=12$. Do stair-step division and read the remainders down.
3. **(C)** Solving for the points of equality using the equations $2x-3=4-x$ and $-(2x-3)=4-x$ yields critical values at $\frac{7}{3}$ and $-1$. Testing intervals revels that the solutions lie between these numbers, and the integers on the interval are 0, 1, and 2.
4. **(D)** Using the Remainder Theorem, $f(-1)=a-b=4a$, so $b=-3a$. The definition of a polynomial dictates that $f(2)-30=0$. Solve $16a+2(-3a)=30$ to find that $a=3$ and $b=-9$.
5. **(B)** Note that 0! = 1, and that $\sqrt{1}<\sqrt{\frac{7}{5}}<\sqrt{4}$. $1-(-2)+1=4$.
6. **(A)** Only the last statement is true.
7. **(D)** For any line $Ax+By=C$, the slope of a perpendicular line is $\frac{B}{A}$.
8. **(A)** Eccentricity = $\frac{c}{a}$ from the ellipse's standard form, so $E^{-2}=\frac{a^2}{c^2}$. $b=\frac{c}{2}$, and

   $c=\sqrt{a^2-b^2}\rightarrow c^2=a^2-\frac{c^2}{4}\rightarrow 5c^2=4a^2\rightarrow \frac{a^2}{c^2}=\frac{5}{4}$.
9. **(C)** Solving $\frac{1}{2}-(-2)=\frac{1}{2}x$, $x=5$.
10. **(C)** The parabola opens away from the $x$-axis, its directrix.
11. **(D)** True for $f(x)=a^x$ whenever $0<a<1$.
12. **(C)** Only one combination of two contestants will satisfy me, and there are ${}_5C_2=\frac{5!}{2!3!}=10$ possible combinations.
13. **(C)** $\left(\frac{2+3i}{1+2i}\right)\left(\frac{1-2i}{1-2i}\right)=\frac{2+3i-4i-6i^2}{1-4i^2}=\frac{8-i}{5}$
14. **(D)** Any positive real number except 1 may be used.
15. **(D)** $3p-rp=-5-t\rightarrow p(3-r)=-(t+5)\rightarrow p=\frac{t+5}{r-3}$
16. **(B)** $f(g(h(1)))=f(g(1+2i))=f((1+2i-2)^2)=f(-3-4i)=|6+4i|=2\sqrt{13}$.

    $f(g(h(4)))=f(g(2+i))=f((2+i-2)^2)=f(-1)=|3+1|=4$.

    $f(g(h(9)))=f(g(5))=f((5-2)^2)=f(3)=|3-9|=6$.
17. **(B)** I is not true, because $f$ has a positive leading coefficient and is an odd degree, so $f(x)$ approaches $-\infty$ as $x$ approaches $\infty$. II and III are both true by Des Cartes' Rule of Signs.
18. **(B)** $9^x=4\rightarrow \log_9 4=x\rightarrow x=\frac{\log 2^2}{\log 3^2}\rightarrow x=\frac{2\log 2}{2\log 3}=\frac{\log 2}{\log 3}=\log_3 2$
19. **(C)** $(a+b)^2=a^2+b^2+2ab=4ab+2ab=6ab$ ; $(a+b)^2=4^2=16$; $6ab=16\rightarrow ab=\frac{8}{3}$
20. **(A)** $10x^2+29x-21=(5x-3)(2x+7)$ so $-3+7=4$.

21. **(B)** $\sqrt[5]{\left(\frac{1}{8}\right)^3} = \sqrt[5]{(2^{-3})^3} = \sqrt[5]{2^{-9}} = \left(2^{-9}\right)^{\frac{1}{5}} = 2^{-\frac{9}{5}}$

22. **(C)** Each year, the amount is multiplied by 1.05. This is normal exponential growth, not continuous growth (modeled by choice A), linear growth (modeled by choice B), or pure nonsense (modeled by choice D).

23. **(C)** $(x-5)^2-(2x-5)^2-x+1=0 \;\rightarrow\; \left(x^2-10x+25\right)-\left(4x^2-20x+25\right)-x+1=0$

$-3x^2+9x+1=0$, and the sum of the solutions is $-\frac{b}{a}=-\frac{9}{-3}=3$.

24. **(A)**

$$\frac{(7)!\left(\frac{14!}{6!8!}\right)}{(10)!}=\frac{7!14!}{6!8!10!}=\frac{\left[(6!)(7)\right]\left[(10!)(11)(12)(13)(14)\right]}{(6!)(8!)(10!)}=\frac{(7)(11)(12)(13)(14)}{(8)(7)(6)(5)(4)(3)(2)}=\frac{(13)(11)(7)}{(8)(5)(3)(2)}=\frac{1001}{240}\approx 4.2$$

25. **(B)** Let $M=a+bi$, so $N=a-bi$. $|a+bi|=|a-bi|=\sqrt{a^2+b^2}$, so $2\sqrt{a^2+b^2}=12 \rightarrow a^2+b^2=36$.
$12=M^2+N^2=(a+bi)^2+(a-bi)^2=a^2+2abi-b^2+a^2-2abi-b^2=2a^2-2b^2$, so $a^2-b^2=6$.
$a^2-b^2+a^2+b^2=42 \rightarrow a^2=21 \rightarrow a=\sqrt{21}$, and by substitution $b=\sqrt{15}$.

$$\left|\frac{M+N}{M-N}\right|^2=\left|\frac{a+bi+a-bi}{a+bi-(a-bi)}\right|^2=\left|\frac{2a}{2bi}\right|^2=\left|\frac{a}{bi}\right|^2=\left|\frac{\sqrt{21}}{i\sqrt{15}}\right|^2=\frac{7}{5}.$$

26. **(C)** $b^2-4ac=0$, so $(2k)^2-4(1)(k^2-9k+9)=0 \rightarrow 4k^2-4k^2+36k-36=0 \rightarrow 36k=36 \rightarrow k=1$.

27. **(C)** $5y^2-4x^2=-20 \rightarrow \frac{x^2}{5}-\frac{y^2}{4}=1$ The hyperbola has a horizontal transverse axis, so the foci are on the $x$-axis. $c=\sqrt{a^2+b^2} \rightarrow c=3$.

28. **(D)** $N\left(\frac{70}{100}\right)\left(\frac{140}{100}\right)\left(\frac{75}{100}\right)=\frac{147}{200}N$

29. **(C)** The equations of the lines are $y=mx+b+13$ and $y=-\frac{x}{m}+b$. Substituting (6, 6) yields $-6m-7=b$ and $6+\frac{6}{m}=b$. $-6m-7=6+\frac{6}{m} \rightarrow 6m^2+13m+6=0 \rightarrow (2m+3)(3m+2)=0 \rightarrow m=-\frac{2}{3},-\frac{3}{2}$.

Case I: $m=-\frac{2}{3}$

$-6m-7=b \rightarrow -6\left(-\frac{2}{3}\right)-7=b \rightarrow b=-3$

$y=mx+b+13$

$y=-\frac{2}{3}x+10$

***x*-intercept: 15**

$y=-\frac{x}{m}+b$

$y=\frac{3}{2}x-3$

***x*-intercept: 2**

distance between = 15 – 2 = 13

Case II: $m=-\frac{3}{2}$

$-6m-7=b \rightarrow -6\left(-\frac{3}{2}\right)-7=b \rightarrow b=2$

$y=mx+b+13$

$y=-\frac{3}{2}x+15$

***x*-intercept: 10**

$y=-\frac{x}{m}+b$

$y=\frac{2}{3}x+2$

***x*-intercept:** $-3$

distance between = $10-(-3)$ = 13

30. **(A)** $f(32)=f(-16)=f(8)=f(-4)=f(2)=f(-1)=2^{-1}$. $f(9)=2^9$. $f(14)=f(-7)=2^{-7}$.
$f(2)=f(-1)=2^{-1}$. $f(40)=f(-20)=f(10)=f(-5)=2^{-5}$. $2^{-1}\cdot 2^9\cdot 2^{-7}\cdot 2^{-1}\cdot 2^{-5}=2^{-5}=\frac{1}{32}$.