Barbara Nunn Test

2005-2006 *no calculator allowed*

SOLUTIONS

B 1. Shortest diagonal is when the rectangle is a square. Side of square is 5, diagonal is $5\sqrt{2}$.

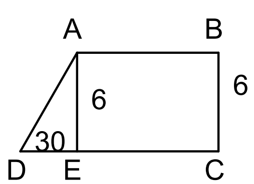
A 2. Adding the 3 expressions gives $2x+2y+2z=36$, $x+y+z=18.$

D 3. x = amount at 6%, 8000-x=amount at 5%; 0.06x+0.05(8000-x)=452;x=5200

C 4. Substituting 2 for *x* gives 2^6-21= 64-21 = 43.

C 5. Multiply 1st equation by 3 which gives $2x-15y=-3$, multiply 2nd equation by 2 gives $5x-14y=16$. Solving this system gives (6,1) and the sum is 7.

D 6. $AB=EC=\sqrt{192}$, $DE=6\sqrt{3}$, $DC=6\sqrt{3}+8\sqrt{3}=14\sqrt{3}$



E 7. $\left(\frac{1}{4}\right) \quad =2^{-2(x-1)}$; $8^{2x-1}=2^{3(2x-1)}$;

Since the bases are the same, set the exponents equal and solve for x.

$-2x+2=6x-3$;x=$\frac{5}{8}$

C 8. $\frac{4}{5}$ Begin simplifying with the last fraction.

A 9. $\frac{xt}{y}+\frac{yt}{x}=x^2+y^2$; $\frac{x^2t+y^2t}{xy}=x^2+y^2$;

$x^2t+y^2t=xy\left(x^2+y^2\right)$;

$t\left(x^2+y^2\right)=xy\left(x^2+y^2\right)$; $t=xy$.

E 10. $(-1+i\sqrt{3})^2=-2-2i\sqrt{3}$;

$(-2-2i\sqrt{3})(-1+i\sqrt{3})=8$

A 11. Longest chord is the diameter Completing the square gives the equation: $(x-4)^2+(y+1)^2=20$,

$r=2\sqrt{5}, d=4\sqrt{5}$

D 12. Drawing the line segment parallel to the side of 12 makes similar triangles by AA. Ratio of areas is $\frac{3}{4}$ so ratio of sides is $\frac{\sqrt{3}}{2}$. Set up the proportion $\frac{\sqrt{3}}{2}=\frac{x}{12}$ makes $x=6\sqrt{3}$.

D 13. $z^2\frac{x}{y}=z+\frac{x}{y}; z^2x=zy+x;$

$\frac{z^2x-x}{z}=zy; zx-\frac{x}{z}=y$

D 14. Edges will be *e,e,e-1*. So the volume will be $e^2(e-1)=100$, $e=5.$

E 15. $1000n=123.\overline{123}$, $n=.\overline{123}$; subtracting these two equations gives $999n=123$, solving for *n* gives the fraction for this repeating decimal. $\frac{123}{999}=\frac{41}{333}$and 41-333=-292.

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A 16. $10101_2 = 1+4+16 = 21_{10}$. Adding 10 gives 31_{10} which is 11111_2.

C 17. $\frac{600}{x} = y, \frac{600}{x+2} - y - 25$. Solving this system gives the quadratic $25x^2 + 50x - 1200 = 0,$ and $x = 6$ or -8, so the number of original kids is 6 and +2=8.

B 18. $\frac{\frac{4}{3}\pi r^3}{r^3} = \frac{4\pi}{3}$

B 19. $x^2 - y^2 = 6k^2; x + y = 2k;$ $(x-y)(x+y) = 6k^2;$ substituting $2k$ for $x+y$ gives $2k(x-y) = 6k^2;$ solving for $x-y$ gives $\frac{6k^2}{2k} = 3k$.

A 20. $\begin{cases} u = 1+2t \\ 3t = 2+u \end{cases}$; solving this system gives $t = 3, u = 7, 7+3 = 10$

C 21. This is one of the hundreds of proofs of the Pythagorean Theorem. The sum of the areas of the smaller triangles equals the area of the larger triangle.

B 22. If 10 is opposite 27 keep working down from 10 to see that 1 is opposite 18. The difference is 17 so 17 is opposite 34 which is the number of students in the class. Since 10 is opposite 27, that difference is also 17 so every difference must be 17.

D 23. Use they altitude on the hypotenuse theorems which use the geometric mean. $4\sqrt{5} = \sqrt{AD(AD+16)};\ AD = 4$. $BC = \sqrt{16 \bullet 20} = 8\sqrt{5}$.

B 24. The median to the hypotenuse is half the hypotenuse so the hyp is 34. The sum of the legs is 46. So the triangle is a multiple of an 8-15-17 right triangle making the legs 16 and 30. $A = \frac{1}{2} \bullet 16 \bullet 30$ which is 240. To find the radius of a circle inscribed in a triangle use the formula Area/semiperimeter = $\frac{240}{40} = 6$. The area of the circle is 36π . So area is $240 - 36\pi.$

C 25. Writing the equation of variation: $i = k\frac{b}{d^2}$. The constant $k = \frac{id^2}{b}$. Substituting both sets of values and setting equal gives $\frac{12 \bullet 4}{6} = \frac{1 \bullet d^2}{8}$. $d = 8$.

C 26. The mode = x = 49, the range = z = 79
the median = y = 81,
the mean = w = 66,
w+2x+3y+4z=723

C 27. $\sqrt{10 \bullet 9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}$ = $\sqrt{5^2 \bullet 7 \bullet 1 \bullet 2^8 \bullet 3^4}$ = $720\sqrt{7}$. $720 = 6!$. $6!\sqrt{7}$.

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D 28. $\log_2(\log_3(\log_4 b)) = 0$; $\log_3(\log_4 b) = 1$;
$\log_4 b = 3; b = 64$;
$\log_4(\log_3(\log_2 a)) = 0$; $\log_3(\log_2 a) = 1$;
$\log_2 a = 3; a = 8$. $a : b = 8 : 64$ which simplifies to $1 : 8$.

C 29. d=20, r=10, so OT (a radius) =10.
Draw the perpendicular from O to TA and label S. OS=x, ST=$\frac{1}{2}x$. Use Pythagorean theorem to solve.
$\left(\frac{1}{2}x\right)^2 + x^2 = 100$; $x^2 = 80$ which is the area of the square.

D 30. $f(x) = 19x + 99$; to find inverse swap x and y, $x = 19y + 99$ and solve for y.
$y = \frac{x - 99}{19}$, $a = \frac{1}{19}, b = -\frac{99}{19}$,
$\frac{1}{\frac{1}{19} - \frac{-99}{19}} = \frac{19}{100}$.