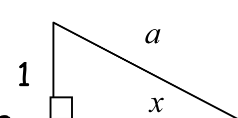
Helen Dostal Test

2005-2006 *no calculator allowed*

Solutions:

1. **D**. a is divisible by 3 and b is divisible by 5, by the first equation. The second tells us that a+b is even and less than 50. a<10 so b<50/3. b=15.
2. **D**. 54 mph times 10 hours gives 540.
3. **C**. $m^2 = 4^{\frac{3}{2}}$ so m= $\sqrt{8}$.
4. **E**. (60) a+b+c=60, and a+b+c+d=120. Subtract to get d=60.
5. **B**. If the base of the larger is 2 by 8 then, without the small box, we see ph=20(10) for lateral faces. Top adds 16, for 216. If the small box has 1 by 1 face down, we now have 215 on the big box. The small box will then add 9 to get 224.
6. **B**. Work outward in, $\log_3 \log_5 x = 2, \log_5 x = 9$ so $x = 5^9$ This has 10 factors.
7. **B**. A quadrilateral has angles total 360 so 360-120-72(ext. angle of the pentagon) gives 168.
8. **A**. If k is positive, then we get -4k since we multiply $i\sqrt{8}\ i\sqrt{2}$. If k is negative we get a positive value, but since k<0, -4k is a positive value.
9. **D**. $\frac{(1-i)}{(a+2i)} \bullet \frac{(a-2i)}{(a-2i)} = \frac{(a-2)+(-2-a)i}{a^2+4}$ and setting a-2=a gives a=1, and so (a+1)i+(a-1)i=2i+0i=2i.
10. **C**. Since e is approximately 2.7, we know $2.7^x = 3$ gives a number greater than 1. pi is about 3. log100=2 and the third term is $\frac{\sqrt{13}}{5}$ which is less than 1. This is the smallest.
11. **D**. $\frac{\frac{3}{2} \bullet -2}{-1-0}$ gives 3.
12. **C**. x=6, x=6(4), x=6(9), x=6(16).
13. **C**. Sums range from 3 to 199. 197 sums
14. **A**. x+3y=180. Try 10+3y=180. Divide each term by 3 to get y, we see 10/3 is not an integer. So x can be 30, 60, 90, 120, 150. y can then have 5 values.
15. **C**. Multiply numerator and denominator by x^2-1 gives $\frac{x-1-x-1}{x+1} = 2$ gives x= -2. So 1-x=3.
16. **C**. $4x = x^4$ and since x cannot be 0, we divide by x to get $x^3 = 4$.
17. **D**. $10 = \sqrt{10x+10}$. Square and solve to get x=9.
18. **D**. A is at (0, 1) and B and C are (3,0) and (-2, 0) to give base is 5 and height is 1. Area is 2.5.
19. **C**. There are 4(3) possible numbers. Out of those, primes are 13, 23, 53, and 31, we get 4/12 to get 1/3.
20. **B**. 1.50X-2/3X = (3/2-2/3)X = (5/6)X
21. **C**. The only integers which will have 3 factors are perfect squares of primes. 4, 9, 25, 49
22. **B**. 12(12)(3)+64n = 12(12)(4). 64n=144(4)-144(3). 64n=144. 2 is too small and 3 too large, so after n=3, the cubes will be covered.
23. A. Use the Pyth. Theorem to get the missing side to be $\sqrt{a^2-1}$. This is also the cotx.



24. **C**. $\frac{5!}{2!2!} = 30$

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25. **D**. The K must be the middle, and to one side we have permutations of ISME, of which there are 4!=24. The other side must match in reverse order, so there are only 24 orders.

26. **B**. (n-5)+(n-4)+(n-3)+(n-2)+(n-1)+n+(n+1)...+(n+5)= k^3 so we get 11n= k^3 so n must be 11(11) or 121. The least number is 121-5=116.

27. **C**. The roots of f are at x=1 and x=-3 so we see what values of f(x) give us those numbers. Draw a horizontal line at x=1 and we get 3 values. Do the same at x= -3 and we get one value. Total 4.

28. **A**. $\sin\theta = \frac{1}{2}\cos\theta\sqrt{3}$ gives $\tan\theta = \frac{\sqrt{3}}{2}$ so $\cos\theta = \frac{2}{\sqrt{7}}$ by using the Pythagorean theorem. Area is then $\frac{1}{4}\left(\frac{2}{\sqrt{7}}\right)^2\sqrt{3}$ which gives choice A.

29. **C**. Group places by two: 11, 11, 13. Convert by 5(1)+1 and 5(1)+3 to get 668.

30. **A**. The sum of the squares of 1 through n give $\frac{n}{6}(n+1)(2n+1)$ and the coefficient fraction is $\frac{1}{n(n+1)}$. Reduce to get each term is $\frac{2n+1}{6}$. This gives the reduced series 1/2, 3/6, 5/6, 7/6, ... which is arithmetic: the sum is $\frac{n}{2}(a_1 + a_n) = \frac{40}{2}(\frac{3}{6} + \frac{81}{6})$ or 20(84)/6=20(42)/3=20(14)=280.