Helen Dostal Test

FAMAT Fall 2003

<u>Solutions:</u>

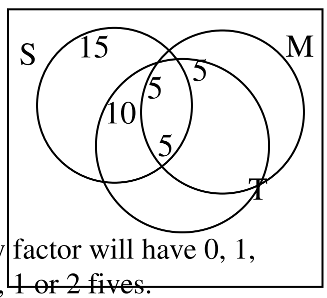
D. 1. $\dfrac{2^{x+1}-2^{x+2}}{4^x}=\dfrac{2^x(2-2^2)}{2^{2x}}=\dfrac{-2}{2^x}=\dfrac{-1}{2^{x-1}}$ which is choice D.

D. 2. To produce an integer, x must be a perfect square whose root must be from 0 to 144. Trying all of these gives 9, 64, 121, 144 and the answer is D.

D. 3. Divide the first inequality by 2 and get $|x+3|>4$ Since the second ineq. says that |x+3| is less than 0, we have only one part of the inequality's usual solution: $x+3<-4$. so $x<-7$. Choice D.

E. 4. i may be true but only if $\overline{AC}$ is the hypotenuse. ii should not be inclusive, by the Triangle Inequality Th. The triangle may be obtuse. None are true.

C. 5. 20 Saturday + 5 only Tuesday+5 other weekdays. The Mondays may also shop on Saturday and likewise Tuesday shoppers. So there are a min of 40. One possible distribution is shown.



D. 6. $200=2^3\bullet 5^2$ so any factor will have 0, 1, 2, or 3 twos, and/or 0, 1 or 2 fives. 4(3)=12 possible factors.

A. 7. This fraction reduces to $\dfrac{-1\bullet 2003}{2004}$ which is between 0 and -1. So [x]= -1

C. 8. Let x be BC: $\dfrac{4}{6}=\dfrac{8}{x}$ gives BC=12. E is the midpoint so EC=half of 6+12, which is 9. So 12-9=3. Choice C.

C. 9. $S=\dfrac{n}{2}(a_1+a_n)=\dfrac{19}{2}(-8+28)=190.$

B. 10. Add the equations: $6x+6y+6z=15$. Divide by 6: x+y+z= 15/6 = 5/2.

A. 11. Each diameter is 16. So we have a triangle (with the diagonal) which is similar to a 3-4-5 triple, times 16. So the diagonal must be 16 times 5 which is 80. Choice A.

A. 12. Since log5+log2=log10=1, $\log\dfrac{x}{10}$=1 and x=100.

B. 13. The reals and nonreals are all a subset of the complex numbers.

D. 14. $\dfrac{1}{3/2}=\dfrac{2}{3}$ and $\dfrac{1}{1+i}=\dfrac{1-i}{2}$ after multiplying numerator and denominator by the conjugate of 1+i. The product is then $\dfrac{2}{3}\bullet\dfrac{1-i}{2}\bullet\dfrac{3}{1}$ is then 1-i. Choice D.

A. 15. At x=0 we get 1. At x=3 we get 4. Since the vertex of the parabola is at x= –b/(2a) we get the vertex is at x=2 and at x=2 we get y=5. So the range is from 1 to 5 inclusive.

D. 16. The sum of the roots of the equation is –b/a which is -4. So 3+m+n= -4. So m+n= -7

B. 17. Since the 2 is the n's digit, and the 5 is the units digit, both are less than n squared. So the number will be 2(n)+5 in base n squared.

A. 18. There are 40 possible numbers from 12 to 98. To be even, we have to finish with a set A number and to be less than 50 we need to begin with a set B number (since we used the set A number for the end. The choices are 1 and 3, so we can have 12,14, 16, 18, 32, 34, 36, 38. The prob is then 8/40 which is 1/5.

C. 19. The perimeter is L+L$\sqrt{3}$+2L where the L is the short leg of the triangle. So L must be 5 and the hypotenuse must be 10.

A. 20. $2\sin x\cos x=\dfrac{1}{3}$ so $2a\sin x=\dfrac{1}{3}$ and $\sin x$ is $\dfrac{1}{6a}$.

D. 21. Choice *i* is false.

B. 22. The vertex angle is R so 30+2(2x-3)=180 which gives x=39.

A. 23. Since $\cos 2x=\cos^2 x-\sin^2 x$ then the equation reduces to $\cos^2 x=\dfrac{1}{4}$ and so the cosine is 1/2 since it is positive.

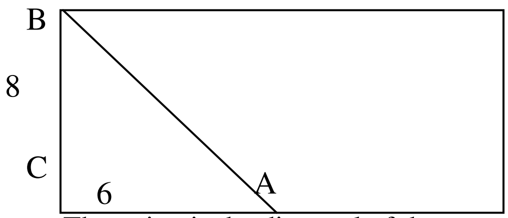
B. 24. We want to fail the first try, so we have $\dfrac{5}{6}\bullet\dfrac{1}{5}=\dfrac{1}{6}$ for the probability. The odds are therefore 1:5. Choice B.

B. 25. $\dfrac{5+x}{50+x}=\dfrac{1}{5}$ since we add x mL of acid to the original 5, and the total, and it must be equal to 20% which is 1/5. This solves to 25/4.

C. 26. y=3x and y=$-\frac{1}{3}x+2$ meets at

$3x=-\frac{1}{3}x+2$ where x is 3/5.

C. 27. Cut the cylinder open from B to C.



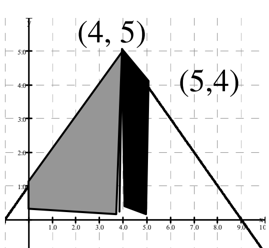
The string is the diagonal of the rectangle and the width is half the circumference of the circle, 6. The diagonal is 10.

C. 28. Using the law of Cosines gives $16+9-2(3)(4)\cos A=(BC)^2$. Since angle A is obtuse, the cosine is negative and so we substitute -11/24 and get BC=6. Choice C.

C. 29. The term is when $(2x^2)$ is raised to the 2 and (-1/x) is raised to the 3.

So we have $C(5,3)(2x^2)^2\left(\frac{1}{x}\right)^3$ which is 10(4)(1)=40 for the coefficient of x.

B. 30.



We have two trapezoids with horizontal heights. The first has area $\frac{1}{2}(4)(1+5)$ which is 12. The second (right hand) has area $\frac{1}{2}(1)(5+4)$ which is 4.5. The total area is 16.5.