team
solutions

1. The sum of the first 100 even natural numbers is

$$S_{100}=\frac{100}{2}(2+200)=50\cdot 202=10100$$, so A = 10100

The sum of the first 100 natural odd numbers is

$$S_{100}=\frac{100}{2}(1+199)=50\cdot 200=10000$$, so B = 10000

$$A-B=10100-10000=100$$

2. In a square of side length 12 inches, the diagonal is $W=12\sqrt{2}$
The area of this square will be $X=144$
The circle inscribed will have a diameter of 12 inches, so the circumference is $Y=12\pi$
The area of this circle will be $Z=36\pi$ since its radius is 6 inches.

$$\frac{W}{X}\bullet\frac{Z}{Y}=\frac{12\sqrt{2}}{144}\bullet\frac{36\pi}{12\pi}=\frac{\sqrt{2}}{12}\bullet\frac{3}{1}=\frac{\sqrt{2}}{4}$$

3. The general equation describing this projectile motion is $h(t)=-16t^2+v_0t+h_0$. We are given that his initial height is 324 and the ball's initial velocity is 64, so our specific equation in this case is $h(t)=-16t^2+64t+320$

A is the time coordinate of the vertex of this parabola. In this equation

$$A=\frac{-64}{2(-16)}=2$$

B is the height coordinate of the vertex of this parabola. In this equation

$$B=-16(2)^2+64(2)+320=-64+126+320=384$$

C is the time it takes to return to its original height. If A is 2, then C is 4.
So our sum A + B + C = 2 + 384 + 4 = 390

4. Here, we want to establish two different dist = rate X time equations. $35(x+4)+60x=995$. IN this equation x represents the time spent on the highway. Solving the linear equation we get x = 9, so the total time is x + x + 4 = 9 + 9 + 4 = 22 hours.

5. The equation to solve here is $x^2+(x+1)^2+(x+2)^2=365\rightarrow 3x^2+6x-360=0$

$$3x^2+6x-360=0\rightarrow x^2+2x-120=0\rightarrow (x+12)(x-10)=0$$

Since a square can't have a negative side length, 10 is the side of the smallest square, so 12 is the side of the largest.

6. Start with the vertex form of the parabola and we have $y = a(x+2)^2 - 3$
Using the point (1,2) we have

$$2 = a(1+2)^2 - 3 \rightarrow 2 = 9a - 3 \rightarrow 5 = 9a \rightarrow \frac{5}{9} = a$$

Now, going back to the vertex form we have

$$y = \frac{5}{9}(x+2)^2 - 3 \rightarrow y = \frac{5}{9}x^2 + \frac{20}{9}x - \frac{7}{9}$$

The sum of A, B and C is 2

7. The equation we need to solve here is
$.25x + .75y = 70 \rightarrow 25x + 75y = 7000 \rightarrow x + 3y = 280$
Since we want whole numbers we can quickly see that the largest possible y value is 93 with x being 1. The smallest y value that would satisfy this equation is 61, so there are 34 solutions (ranging from value that would satisfy this equation is 61, so there are 34 solutions (ranging from $y = 61$ to $y = 93$ including both endpoints)

8. We need to use the rational root theorem to break down these functions. The function f reduces to $f(x) = (x-1)(x+2)(x^2-3)$ We now only need to see if either of the linear factors of f are also factors of g. They each are so the common linear factors are $x - 1$ and $x + 2$.

9. The quickest way to check this is to concentrate on the largest divisor, which is 7. Numbers that have a remainder of 5 when divided by 7 are 12, 19, 26, 33, 40 etc. The first such number that satisfies the other conditions is 68.

10. Here we have $a = 20$ and $c - b = 8$. By Pythagoras we know

$$a^2 + b^2 = c^2 \rightarrow a^2 = c^2 - b^2 \rightarrow a^2 = (c-b)(c+b)$$

$$400 = 8(c+b) \rightarrow 50 = c + b$$

So, now we know that the sum of b and c is 50 while their difference is 8. The numbers satisfying this are 29 and 21 so the hypotenuse is 29.

11. The x-intercepts occur when $y = 0$ so we have
$4x^2 - 16x - 32 = 0 \rightarrow x^2 - 4x - 8 = 0$

$$x = \frac{4 \pm \sqrt{48}}{2} \rightarrow \frac{4 \pm 4\sqrt{3}}{2} \rightarrow 2 \pm 2\sqrt{3}$$

The y-intercepts are when $x = 0$ so we have
$16y^2 + 32y - 32 = 0 \rightarrow y^2 + 2y - 2 = 0$

$$y = \frac{-2 \pm \sqrt{12}}{2} \rightarrow y = \frac{-2 \pm 2\sqrt{3}}{2} \rightarrow y = -1 \pm \sqrt{3}$$

Completing the square twice shows that the center of the ellipse is at the point (2, -1) so for the sum of all these values we have

$$2 + 2\sqrt{3} + 2 - 2\sqrt{3} - 1 + \sqrt{3} - 1 - \sqrt{3} + 2 - 1 = 3$$

12. For Beth we have $3t+5t=15 \to 8t=15 \to t=\frac{15}{8}$ where t is the time spent for each part of the race, so her total time is 3 hours 45 minutes. For Doris we have her running equation as $7.5=5t_r \to 1.5=t_r$ and her walking equation as $7.5=3t_w \to 2.5=t_w$ so her total time is 4 hours and Beth wins the race by 15 minutes.

13. Call the two numbers x and y. We have the following equations to solve: $x+y=18,\ x^2-y^2=72$. In the quadratic, we see that the expression factors so our system reduces as follows:
$x^2-y^2=72 \to (x+y)(x-y)=72 \to 18(x-y)=72 \to x-y=4$. So now we have

$$\begin{aligned} x + y &= 18 \\ x - y &= 4 \end{aligned}$$

The solutions are $x=11,\ y=7$. So the final answer is that the product is 77.

14. $\frac{x}{x+2}+\frac{x+2}{x}=2x \to \frac{x^2+(x+2)^2}{x(x+2)}=2x \to x^2+x^2+4x+4=2x^3+4x^2$

$0=2x^3+2x^2-4x-4 \to 0=(2x^2-4)(x+1)$

The only rational solution is $x=-1$

15. $x^2-\frac{1}{x^2}=10 \to \left(x^2-\frac{1}{x^2}\right)^2=100 \to x^4-2+\frac{1}{x^4}=100 \to x^4+\frac{1}{x^4}=102$