Answer Key

1. D	6. D	11. C	16. A	21. C	26. B
2. B	7. B	12. D	17. B	22. D	27. B
3. A	8. A	13. A	18. C	23. B	28. B
4. D	9. D	14. B	19. A	24. E	29. A
5. A	10. C	15. E	20. A	25. B	30. A

Solutions

1. Expanding the left side results in $-3x + 6 + 2x + 10 - 4x + 4 = 10$. Combining like terms yields $-5x + 20 = 10$, which results in $x = 2$.

2. $x = 1$ is clearly a solution. Factoring this out yields $(x - 1)(x^2 - 2x - 2) = 0$. The solutions to $x^2 - 2x - 2 = 0$ are complex, therefore the sum of the *real* solutions is 1.

3. $g(2) = 2$, $f(2) = 6$, and $g(6) = 35$. $f(g(2)) - g(f(2)) = f(2) - g(6) = 6 - 34 = -28$

4. Squaring $x - y$ yields $(x - y)^2 = x^2 - 2xy + y^2 = 16$. From this, we find that $xy = -1$. $x^{-1} - y^{-1}$ is equivalent to $\dfrac{y - x}{xy}$ which is 4.

5. Putting the equation into the standard form of an ellipse, we get: $\dfrac{(x-1)^2}{9} + \dfrac{(y-3)^2}{4} = 1$. From this, we see that $a^2 = 9$ and $b^2 = 4$. The area of an ellipse is $ab\pi$, so the area is 6π.

6. The discriminant of a quadratic equation is $b^2 - 4ac$. In this case, $a = 1$, $b = -4$, and $c = -32$. $16 + 128 = 144$.

7. This can be rewritten as $\frac{1}{2}\log_3(x + 1) = \log_3 x$, or $x + 1 = x^2$. This has two solutions, $x = \dfrac{1 \pm \sqrt{5}}{2}$. However, negative numbers are not in the domain of the log function. Therefore, there is only one solution: $x = \dfrac{1 + \sqrt{5}}{2}$.

8. Completing the square and putting the equation into the standard form of a parabola yields: $f(x) = (x + 2)^2 - 7$. The vertex of this parabola is at $(-2, -7)$, and it opens upward. Therefore, the minimum value is -7.

9. If $2x + 3 < 0$, then the inequality can be written as: $-(2x + 3) \leq 11$, or $x \geq -7$. If $2x + 3 \geq 0$, then the inequality is $2x + 3 \leq 11$, or $x \leq 4$. Combining the two inequalities gives: $-7 \geq x \leq 4$. There are 12 integers between -7 and 4, inclusive.

10. Diving the equation by 64 puts the equation in standard form. We see that $a^2 = 16$ and $b^2 = 4$. The length of the major axis is $2a = 8$ and the length of the minor axis is $2b = 4$. Their sum is 12.

11. The first inequality represents the area within the square with vertices at $(4, 0)$, $(-4, 0)$, $(0, 4)$ and $(0, -4)$. The second inequality represents the area outside the circle centered at $(4, 0)$ with radius 4. The area represented by both inequalities is the area of the square minus the overlapping quarter of the circle. $32 - 4\pi$

12. Moving the two over to the left, and then factoring results in: $(x+2)(x+1)(x-1) \leq 0$. x to the left of -2 results in negative values. x between -2 and -1 results in positive values. x between -1 and 1 results in negative values, and x to the right of 1 results in positive values. Therefore, the solution set is: $(-\infty, -2] \cup [-1, 1]$.

13. Multiplying both sides by $x^2 - x - 2$ yields: $A(x-2) + B(x+1) = 2x + 3$. Setting $x = 2$, it is clear that $B = 7/3$. By setting $x = -1$, we see that $A = -1/3$. Subtracting, $A - B = -8/3$.

14. The numerator factors to $(x-1)^2(x+1)$ and the denominator factors to $(x-1)(x+2)(x+1)$. Cancelling out $(x-1)$ and $(x+2)$ leaves $\dfrac{(x-1)}{(x+2)}$. There is a vertical asymptote at $x = -2$. Since the degree of the numerator is equal to the degree of the numerator, we take the ratio of the leading coefficients to find the horizontal asymptote, $y = 1$. There are two asymptotes, $x = -2$ and $y = 1$.

15. Using the zeros of the left column, we find the determinant to be $(x-1)(x-2)(x-3)$. This will be zero for $x = 1, 2, 3$. The sum is 6.

16. The equation factors to: $(x-1)^2(x-2)^2$. The distinct roots are 1 and 2, whose sum is 3.

17. Moving all the terms to one side and rearranging gives: $x^2 - y^2 - x + y$=0. This can factor as follows: $x^2 - y^2 - x + y = (x-y)(x+y) - (x-y) = (x-y)(x+y-1) = 0$. The equation holds true for $x = y$ and $x + y = 1$. These are two lines.

18. According to the Rational Root Theorem, a rational root a/b is a possible root of a function $f(x)$ if a is a factor of the constant term of the function and if b is a factor of the leading coefficient. 3/5 is not a possible root because 3 is not a factor of 10.

19. If $f\left(\frac{1}{x}\right) = \dfrac{x}{x+1}$, then $f(x) = \dfrac{1/x}{1/x+1}$. Simplifying this, we get $f(x) = \dfrac{1}{1+x}$. To find $f^{-1}(x)$, we let $f(x) = x$ and $x = f^{-1}(x)$ to get $x = \dfrac{1}{1+f(x)}$. Solving for $f^{-1}(x)$ yields $f^{-1}(x) = \dfrac{1-x}{x}$.

20. We can find a point on one of the lines, and then use the point-to-line formula. A point on $3x - 4y = 5$ is $(3, 1)$. From the formula, $D = \left|\dfrac{3(3) - 4(1) - 8}{\sqrt{3^2 + 4^2}}\right| = \dfrac{3}{5}$.

21. Let $m = e^x$ to get: $m^3 - 5m^2 + 6m - 2 = 0$. The product of the roots of this equation is 2. If the roots are m_1, m_2, and m_3, then their product will be $m_1 \cdot m_2 \cdot m_3 = e^{x_1} \cdot e^{x_2} \cdot e^{x_3} = e^{x_1+x_2+x_3} = 2$. Thus, $x_1 + x_2 + x_3 = \ln 2$.

22. Squaring both sides gives: $x - \sqrt{x - \sqrt{x - \cdots}} = 100$. Using our original equation, we get $x - 10 = 100$. Clearly, $x = 110$.

23. In a triangle, the sum of two side lengths must be greater than the third side length. Using this, we make the following three inequalities:

$$\begin{aligned} (3x+2) + (5x-1) &> x+3 \\ (5x-1) + (x+3) &> 3x+2 \\ (3x+2) + (x+3) &> 5x-1 \end{aligned}$$

Solving for x in each inequality gives: $x > 2/7$, $x > 0$, and $x < 6$. Combining these three inequalies results in $2/7 < x < 6$.

24. The equation factors as $f(x) = (x+1)(x-1)(x-2)(x-3)$. The largest root is 3.

25. Arnold is three times as old as Bart, so $A = 3B$. Three years ago, Arnold was four times as old as Bart, so $(A-3) = 4(B-3)$. Solving this system results in $B = 9$.

26. The matrix equation represents a system of equations: $x-2y-z = 2$, $2x+3y+3z = 1$, and $x-y-z = 3$. Subtracting the first equation from the third results in $y = 1$. We then have the system: $x - z = 4$ and $2x + 3z = -2$. We get $x = 2$ and $z = -2$. They sum to 1.

27. $a_1 = 1$, so $a_{21} = a_1 + 20d = 121$. The difference is $d = 6$. We want to find the sum of the first 11 terms, so $S = \frac{11}{2}(a_1 + a_1 + 10d) = \frac{11}{2}(62) = 341$.

28. Note that $\left(x^n + \dfrac{1}{x^n}\right)^2 = x^{2n} + \dfrac{1}{x^{2n}} + 2$. Moving the 2 over in our original equation gives $x^8 + \dfrac{1}{x^8} + 2 = 47^2$, or $\left(x^4 + \dfrac{1}{x^4}\right)^2 = 47^2$. From this, we get $x^4 + \dfrac{1}{x^4} = 47$. Adding 2 to both sides, we get $\left(x^2 + \dfrac{1}{x^2}\right)^2 = 7^2$, so $x^2 + \dfrac{1}{x^2} = 7$. Similarly, we find $x + \dfrac{1}{x} = 3$. To find $x^3 + \dfrac{1}{x^3}$, we use the product: $\left(x^2 + \dfrac{1}{x^2}\right)\left(x + \dfrac{1}{x}\right) = x^3 + x + \dfrac{1}{x} + \dfrac{1}{x^3} = x + \dfrac{1}{x} + 3 = 21$. It follows that $x + \dfrac{1}{x} = 18$.

29. y is inversely proportional to x and directly proportional to z, so we can write: $y = (zk)/x$. Substituting the given values of x, y and z results in $k = 4$. Using $y = 10$ and $z = 5$ results in $x = 2$.

30. To find the inverse, set $f(x) = x$ and $x = f^{-1}(x)$ to get $x = \dfrac{3}{4 + 5f^{-1}(x)}$. Solving for $f^{-1}(x)$, we get $f^{-1}(x) = \dfrac{3 - 4x}{5x}$.