

For all questions, answer (E) NOTA means “None of the Above Answers are Correct”
Also, let $\sqrt{-1} = i$ and let \mathbb{R} be the set of Real Numbers.

(1) What is the shape of the graph $r = 16 \sin \theta$?

- (A) circle (B) ellipse (C) cardioid (D) limicon (E) NOTA

(2) Evaluate: $\int_1^2 x^2 e^{2x} dx$

- (A) $\frac{5e^4}{4} - \frac{e^2}{4}$ (B) $\frac{5e^4}{4}$ (C) $\frac{5e^4}{4} + \frac{e^2}{4}$ (D) $\frac{7e^4}{4} - \frac{3e^2}{4}$ (E) NOTA

(3) What is the radius of convergence of the Taylor series expansion centered at $x = 0$ that represents $f(x) = \frac{e^x \sin x}{(x+3)(x-5)}$?

- (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) NOTA

(4) What is the 5th derivative of $f(x) = \arctan x$ at $x = 0$?

- (A) 0 (B) 3 (C) 5 (D) 24 (E) NOTA

(5) What is the volume of $f(x) = \frac{1}{x}, x \geq 1$ rotated about the x-axis?

- (A) π (B) $\frac{\pi}{2}$ (C) 1 (D) $\frac{1}{4}$ (E) NOTA

(6) Evaluate: $\int_1^5 \sin^2(2^3 y) + \cos^2((\log_3 6561)y) dy$

- (A) $\frac{1}{6}$ (B) $-\frac{1}{6}$ (C) 4 (D) -4 (E) NOTA

(7) Find the area of two of the ‘petals’ of $r = \cos 2\theta$

- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{16}$ (E) NOTA

- (8) Find $\frac{d^2y}{dx^2}$ if $x = t^2, y = 2t + 5$
- (A) $-\frac{1}{2t^3}$ (B) $-\frac{1}{t^2}$ (C) 0 (D) $2t$ (E) NOTA
- (9) Which of the following are true?
- I) On a continuous closed curve, the derivative at every point exists and is unique.
- II) If the area under a curve does not converge, then the volume of the curve rotated about the x-axis also does not converge.
- (A) I (B) II (C) I,II (D) None (E) NOTA
- (10) Evaluate: $\int \sin(\sqrt{x}) dx$
- (A) $-\sqrt{x} \cos \sqrt{x} + \sin \sqrt{x} + C$ (B) $\sqrt{x} \cos \sqrt{x} - \sin \sqrt{x} + C$
(C) $-2(\sqrt{x} \cos \sqrt{x} - \sin \sqrt{x}) + C$ (D) $-2(\sqrt{x} \cos \sqrt{x} - \cos \sqrt{x}) + C$ (E) NOTA
- (11) What value does $-\frac{1}{3} + \frac{2}{9} - \frac{1}{9} + \frac{4}{81} - \frac{5}{81} + \frac{6}{81} \dots$ converge to?
- (A) $\frac{3}{16}$ (B) $-\frac{3}{16}$ (C) $\frac{5}{16}$
(D) 0 (E) NOTA
- (12) $\int_0^1 \frac{1}{x^3 + 2x^2 + x + 2} dx = a \ln 3 + b\pi \ln e + c \ln 2$ What is $a + b + c$?
- (A) $\frac{5}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{8}$ (D) 0 (E) NOTA
- (13) Find the slope of the tangent line when $\theta = \frac{\pi}{3}$ of $r = 1 + \sin \theta$
- (A) undefined (B) 0 (C) -1 (D) 1 (E) NOTA
- (14) What is the square of the amplitude of $f(x) = 4 \sin(45x) + 19 \cos(45x)$?
- (A) 377 (B) 415 (C) 81 (D) 23 (E) NOTA

- (15) Find the area of the region bounded by $y = e^x$, $y = x$, $x = 0$ and $x = 1$
- (A) $e - \frac{1}{2}$ (B) $e - \frac{3}{2}$ (C) $e - 1$ (D) $e - \frac{5}{2}$ (E) NOTA
- (16) Evaluate: $\int_0^\pi \sec^2 x \, dx$
- (A) 0 (B) 1 (C) $\frac{\sqrt{3}}{3}$ (D) $\sqrt{3}$ (E) NOTA
- (17) $f(x) = \frac{x^2 \cos(x) + x^4 + 3}{3 - \sin^2(x)}$ can be represented by a Taylor series centered at $x = 0$.
What is the coefficient of the x^9 term of the series?
- (A) $\frac{28}{(451)(9!)}$ (B) $\frac{61}{(345)(9!)}$ (C) $\frac{28}{451}$ (D) $\frac{28}{345}$ (E) NOTA
- (18) Set $S = \{\text{All circles with radii } r, 1 \leq r \leq 5\}$. What is the average area of a circle in S ?
- (A) 9π (B) $\frac{31\pi}{3}$ (C) 13π (D) 31π (E) NOTA
- (19) Find the tangent line at $(0, \frac{1}{2})$ of the cardioid $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{4}$ (D) 1 (E) NOTA
- (20) Evaluate: $\lim_{x \rightarrow \infty} \log\left(\frac{e^{\frac{1}{x}} + 2x^2}{x^2 + 4x}\right)$
- (A) 2 (B) 1 (C) $\log 2$ (D) DNE (E) NOTA
- (21) Find the volume of the polygon with vertices $(1,1), (2,2), (-1,5)$ rotated about the line $x = 7$
- (A) 18π (B) 19π (C) 36π (D) 38π (E) NOTA

- (22) The area of the region that lies inside $r = 3 \sin \theta$ and outside $r = 1 + \sin \theta$ is equal to which of the following integrals?

(A) $.5 \int_{\pi/6}^{5\pi/6} (2 \sin \theta - 1) d\theta$ (B) $\int_{\pi/6}^{5\pi/6} (2 \sin \theta - 1) d\theta$
 (C) $\int_{\pi/6}^{\pi/2} (8 \sin^2 \theta - 1 - 2 \sin \theta) d\theta$ (D) $\int_{\pi/6}^{5\pi/6} (8 \sin^2 \theta - 1 - 2 \sin \theta) d\theta$ (E) NOTA

- (23) The length of one arch of the cycloid ($0 \leq \theta \leq 2\pi$) is equal to which of the following integrals?

$$x = 2(\theta - \sin \theta)$$

$$y = 2(1 - \cos \theta)$$

(A) $\int_0^{2\pi} \sqrt{2(1 - \cos \theta)} d\theta$ (B) $2 \int_0^{2\pi} \sqrt{2(1 - \cos \theta)} d\theta$
 (C) $2 \int_0^{\pi} \sqrt{2(1 - \cos \theta)} d\theta$ (D) $\int_0^{2\pi} \sqrt{2(1 - \sin \theta)} d\theta$ (E) NOTA

- (24) Given that $\int \frac{\sqrt{9 - x^2}}{x^2} dx = A \frac{\sqrt{B + Cx^D}}{x^E} - \arcsin \frac{x}{3} + \mathbf{C}$.

What is $A + B + C + D + E$?

(A) 9 (B) 10 (C) 11 (D) 12 (E) NOTA

- (25) Evaluate: $\lim_{n \rightarrow \infty} \sqrt{\frac{1}{n^3}} + \sqrt{\frac{2}{n^3}} + \sqrt{\frac{3}{n^3}} + \sqrt{\frac{4}{n^3}} + \dots$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{5}$ (D) $\frac{3}{2}$ (E) NOTA

(26) Solve the following Differential Equation by multiplying by e^{x^3} :

$$y' + 3x^2y = 6x^2$$

- (A) $y = \frac{e^{x^3}}{2} + C$ (B) $y = 2 + \frac{C}{e^{x^3}}$ (C) $y = \frac{e^{x^3}}{3}$
(D) $y = \frac{e^{x^3}}{3} + \frac{C}{(x^3)}$ (E) NOTA

(27) Evaluate: $\int_0^{\frac{\pi}{6}} \cos 3x \sin x \, dx$

- (A) $-\frac{3}{8}$ (B) $-\frac{1}{16}$ (C) $\frac{3}{8}$ (D) $\frac{1}{16}$ (E) NOTA

(28) For $|x| < 4$, the series representation of $\frac{4}{4+x}$ is?

- (A) $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{4^{n+1}}$ (B) $\sum_{n=0}^{\infty} (-1)^n \frac{4^n}{x^{n+1}}$ (C) $\sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$ (D) Does Not Converge
(E) NOTA

(29) Which of the following are true?

I) A function can cross its horizontal asymptote

II) If $\lim_{x \rightarrow \infty} f(x) = k$, then $f(x)$ has a horizontal asymptote at $y = k$

- (A) I (B) II (C) I,II (D) None (E) NOTA

(30) Evaluate: $\frac{\pi}{180} \int_0^{30^\circ} \cos x \, dx$

- (A) $\frac{\pi}{360}$ (B) $-\frac{\pi}{360}$ (C) $\frac{\pi\sqrt{3}}{360}$ (D) $-\frac{\pi\sqrt{3}}{360}$ (E) NOTA