

1. [E] $\tan \frac{3\pi}{2} = \frac{\sin \frac{3\pi}{2}}{\cos \frac{3\pi}{2}} = \frac{-1}{0}$. The value is undefined.
2. [B] Keep adding 360° to find a coterminal angle. $\sin(-400^\circ) = \sin(-40^\circ) = -\sin(40^\circ)$ (remember, the sine function is odd.) Referring to the table, $-\sin(40^\circ) = -0.6428$.
3. [B] Converting to degrees, $\frac{\pi}{10} = \frac{x}{180^\circ} \rightarrow x = 18^\circ$. Of course, the cofunction identity for sine states that $\cos(18^\circ) = \sin(90^\circ - 18^\circ) = \sin(72^\circ)$. Referring to the table, $\sin(72^\circ) = 0.9511$.
4. [A] By Euler's formula, $\text{cis } \theta = e^{i\theta}$. So, $\text{cis}(x+y) = e^{i(x+y)} = e^{xi+yi} = (e^{xi})(e^{yi}) = \text{cis}(x)\text{cis}(y)$.
5. [C] Since $\sin(2x) = 2\sin(x)\cos(x)$, it is negative if either $\sin x < 0$ or $\cos x < 0$. We know that $\sec x > 0 \rightarrow \cos x > 0$ and $\csc x < 0 \rightarrow \sin x < 0$. The double cosine identities are irrelevant here because if $x = -45^\circ$, then the conditions given apply but $\cos(2x) = 0$.
6. [C] Rewriting, $y = -2\sin(3[x+2]) + 3$. The amplitude is $|-2| = 2$. The period is $\frac{2\pi}{3}$. $\left|2(2) + 3\left(\frac{2}{3}\right)\right| = 6$.
7. [B] The area of the triangle $= \frac{1}{2}(6)(10)(\sin 58^\circ) = 30\sin(58^\circ)$. Use the table. $30\sin(58^\circ) = 30(0.848) = 25.44$.
8. [A] $\sin^{-1}\left(\sin\left(\frac{5\pi}{3}\right)\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$
9. [A] Converting to a fraction, $\csc \lambda = 2.6 = \frac{26}{10} = \frac{13}{5}$. Then $\sin \lambda = \frac{5}{13}$ and $\cos \lambda = \frac{12}{13}$. So $\tan \lambda = \frac{\sin \lambda}{\cos \lambda} = \frac{5}{12}$.
10. [B] Since the angles of a triangle sum to 180° , $m\angle C = 61^\circ$. Using the Law of Sines, $\frac{\sin A}{a} = \frac{\sin C}{c}$. $\frac{\sin 37^\circ}{a} = \frac{\sin 64^\circ}{10}$ Using the table, then $\frac{0.6018}{a} = \frac{0.8988}{10}$. Cross-multiplying, and rounding to 2 decimal places, $0.9a = 6 \rightarrow 9a = 60 \rightarrow a = 6.\bar{6}$, which rounds to 7.
11. [A] Manipulating the identity $1 + \tan^2 \theta = \sec^2 \theta$, $1 - \sec^2 \theta = -\tan^2 \theta$.
12. [C] Factoring, $(2\sin x + 1)(\sin x + 1) = 0$. The first factor solves to $x = -\frac{1}{2}$ and the second solves to $\sin x = -1$. The possible values are $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$. The sum is $\frac{9\pi}{2}$, and the mean is $\frac{9\pi}{2} \div 3 = \frac{3\pi}{2}$.
13. [D] $\tan \theta = \frac{y}{x}$, so $y = x \tan \theta$.
14. [C] $\sin(3x) = \sin(2x+x) = \sin(2x)\cos(x) + \cos(2x)\sin(x) = 2\sin x \cos^2 x + (1-2\sin^2 x)(\sin x) = 2\left(\frac{2}{5}\right)\left(\frac{\sqrt{21}}{5}\right)^2 + \left(1-2\left(\frac{2}{5}\right)^2\right)\left(\frac{2}{5}\right) = \left(\frac{4}{5}\right)\left(\frac{21}{25}\right) + \left(\frac{2}{5}-2\left(\frac{2}{5}\right)^2\right) = \frac{84}{125} + \left(\frac{50}{125}-\frac{16}{125}\right) = \frac{118}{125}$.

15. [C] First, determine that the triangle is not degenerate. Since $\sqrt{5}$ is between 2 and 3, then $3\sqrt{5}$ is between 8 and 12. Then $4\sqrt{3} = 8\frac{\sqrt{3}}{2} = 8\sin(60^\circ) \approx 6.93$. So 12 is the largest side, and the smaller sides have a sum large enough to guarantee a triangle.
- Second, the Pythagorean Inequality Theorem states that a triangle with sides a , b , and c where $a \leq b \leq c$ is obtuse iff $a^2 + b^2 < c^2$; right iff $a^2 + b^2 = c^2$; and acute iff $a^2 + b^2 > c^2$. Since $(4\sqrt{3})^2 + (3\sqrt{5})^2 = 48 + 45 = 93$ is less than $12^2 = 144$, the triangle is obtuse.
16. [B] The range is $[0, \pi]$ and the domain is $[-1, 1]$. The union is all numbers that lie in either or both intervals, and is $[-1, \pi]$.
17. [A] Law of Cosines! $6^2 = 5^2 + 9^2 - 2(5)(9)\cos J \rightarrow -70 = -90\cos J \rightarrow \cos J = \frac{7}{9}$. There are no cosines on the table, but use your cofunction knowledge. Note from the table that $\sin^{-1}\left(\frac{7}{9}\right) \approx \sin^{-1}(0.778)$ is between 51° and 52° . So $\cos^{-1}\left(\frac{7}{9}\right)$ is between 38° and 39° .
18. [E] Since $\sec \alpha$ is positive, α is in Quadrant IV. $\cos \alpha = \frac{4}{5}$ and $\sin \alpha = -\frac{3}{5}$. Similarly, β is in Quadrant II. So $\cos \beta = -\frac{4}{5}$. $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = \left(\frac{4}{5}\right)\left(-\frac{4}{5}\right) + \left(-\frac{3}{5}\right)\left(\frac{3}{5}\right) = -1$
19. [C] $\sin(x) \geq \frac{1}{2}$ only on the interval $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$, which is $\frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$ units long. The probability that a value is greater than or equal to $\frac{1}{2}$ is $\frac{\frac{2\pi}{3}}{2\pi} = \frac{1}{3}$, so the probability that the value is less than $\frac{1}{2}$ is $1 - \frac{1}{3} = \frac{2}{3}$.
20. [C] Note that $\sin(2x) = \frac{24}{25} \rightarrow \cos(2x) = \frac{7}{25} \rightarrow \cos^2(x) - \sin^2(x) = \frac{7}{25}$. Let's square it.
- $$\cos^4(x) - 2\sin^2(x)\cos^2(x) + \sin^4(x) = \frac{49}{625}$$
- Now, note that
- $$\sin(2x) = \frac{24}{25} \rightarrow 2\sin x \cos x = \frac{24}{25} \rightarrow \sin x \cos x = \frac{12}{25} \rightarrow \sin^2(x)\cos^2(x) = \frac{144}{625}$$
- Substituting,
- $$\cos^4(x) + \sin^4(x) - 2\left(\frac{144}{625}\right) = \frac{49}{625}, \text{ and then } \sin^4(x) + \cos^4(x) = \frac{337}{625}.$$
21. [D] 42'01" is very very slightly more than 42 minutes, which is very very slightly more than $\frac{42}{60} \approx 0.7^\circ$

$$22. \text{ [A]} \frac{(1 + \cos y)(1 - \cos y)}{\tan y \csc y - \cos y} = \frac{(1 - \cos^2 y)}{\frac{\cancel{\sin y}}{\cos y} \cdot \frac{1}{\cancel{\sin y}} - \cos y} = \frac{1 - \cos^2 y}{\frac{1}{\cos y} - \frac{\cos^2 y}{\cos y}} =$$

$$\frac{1 - \cos^2 y}{\frac{1}{\cos y}} = \frac{\cancel{1 - \cos^2 y}}{1} \cdot \frac{\cos y}{\cancel{1 - \cos^2 y}} = \cos y$$

23. [D] Converting to degrees, $2 \operatorname{cis}\left(\frac{3\pi}{15}\right) = 2 \operatorname{cis}(36^\circ) = 2 \cos(36^\circ) + 2i \sin(36^\circ)$, which by cofunction identity is $2 \sin(54^\circ) + 2i \sin(36^\circ)$. Referring to the table, this is approximately $1.6 + 1.2i$, so $|a + 2b|$ is approximately 4.

24. [B] Let's change both graphs to a positive sine with a shift. $-\sin(x - \pi) = \sin(x)$, and

$$\cos\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} + \left(\frac{\pi}{2} - x\right)\right) = \sin(-x) = \sin(x - \pi).$$

25. [D] The dodecagon can be divided into 24 congruent right triangles. In each, the angle at the center is $\frac{360^\circ}{24} = 15^\circ$, and the angle at the vertex is 75° . The short side of the triangle is $(0.5x)$ and to find the height the trig ratio $\tan(75^\circ) = \frac{h}{0.5x}$ is solved. $h = 0.5 \tan(75^\circ)$. The area of the triangle is $\left(\frac{x^2}{8}\right) \tan 75^\circ$. There are 24 of these triangles comprising the dodecagon, so the total area is $3x^2 \tan 75^\circ$.

26. [C] The radicand is read whenever $\sin x \geq \cos x$. Since $\sin x = \cos x$ only at the values $\frac{\pi}{4}$ and $\frac{5\pi}{4}$, then plot those values on a number line and test the intervals. The solution set lies between (and includes) them.

$$27. \text{ [A]} \tan(2\beta) = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{2\left(\frac{12}{5}\right)}{1 - \left(\frac{12}{5}\right)^2} = \frac{\frac{24}{5}}{\frac{25}{25} - \frac{144}{25}} = \left(\frac{24}{5}\right)\left(-\frac{25}{119}\right) = -\frac{120}{119}$$

28. [C] I is not true, because sine is an odd, not an even, function. II is true. Since $0 \leq |\cos x| \leq 1$ for all x , squaring $\cos(x)$ will not increase its value. III is true because $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + \sin(2x)$.

29. [C] The amplitude of either function is irrelevant, since there is no vertical shift involved. The graphs of sine and cosine intersect twice on the interval; doubling the frequency of cosine will double the number of intersection points.

30. [D] Looking at the first six terms, $\frac{1}{2} - \frac{1}{2} - 1 - \frac{1}{2} + \frac{1}{2} + 1 = 0$. After this, the period repeats. Dividing 2007 by

$$6 \text{ and looking at the remainder, only three terms remain after the final period. } \left(\frac{1}{2} - \frac{1}{2} - 1\right)^2 = 1$$