Unless otherwise specified, equations are to be solved over the real numbers.

$\log x$ is the base-10 logarithm; $\ln x$ is the natural (base e) logarithm; $\lfloor x \rfloor$ represents the least integer greater than or equal to x; $a \bmod b$ represents the remainder when a is divided by b.

Answer choice (E) NOTA means "none of these answers."

1. Simplify for $x > 0$: $8^{\log_2 x}$

   (A) $3x$ (B) $4x$ (C) x^3 (D) x^4 (E) NOTA

2. If $\log_a b = 15$, then what is the value of $\log_{a^3} b^5$?

   (A) $\sqrt[5]{15^3}$ (B) 9 (C) 25 (D) $\sqrt[3]{15^5}$ (E) NOTA

3. Solve for x: $\ln(\ln(\ln x)) = 0$

   (A) $x = 1$ (B) $x = e$ (C) $x = e^e$ (D) $x = e^{e^e}$ (E) NOTA

4. Let x be a positive integer. Which of the following represents the number of digits in x?

   (A) $\lfloor \log x \rfloor$ (B) $\lfloor \log x \rfloor + 1$ (C) $\log \lfloor x \rfloor$ (D) $\log \lfloor x \rfloor + 1$ (E) NOTA

5. Simplify: $(e^{\cos x})^2 \left(e^{\sin x}\right)^2$

   (A) 1 (B) e (C) e^2 (D) e^x (E) NOTA

6. Evaluate: $\prod_{n=2}^{\infty} e^{\frac{1}{2^n}}$

   (A) 1 (B) $\sqrt{e}$ (C) e (D) e^2 (E) NOTA

7. Find the 4^{th} root of $4^{14} + 9^{10} + 3^{11}8^5 + 27^5 2^9 + 3^5 2^{23}$.

   (A) 115 (B) 307 (C) 371 (D) 31104 (E) NOTA

8. Find the numerical value of $\log_{\frac{1}{243}}(27\sqrt[4]{27})$.

   (A) $-\frac{15}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{3}{4}$ (D) $\frac{15}{4}$ (E) NOTA

9. Let the range of Arcsin x be $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and let $S(x)$ denote the Shuaian logarithm of x, which is defined as $\log_\pi x$. Solve for x: $\frac{1}{\ln 2}[1 - S(\text{Arcsin } x)] = 2S(e)$

   (A) $x = -\frac{1}{2}$ (B) $x = \frac{1}{2}$ (C) $x = \frac{\sqrt{2}}{2}$ (D) $x = \frac{\sqrt{3}}{2}$ (E) NOTA

10. Solve for x: $2^{6x} + 4^{9+3x} = 4 + 2^{20}$

    (A) $x = \frac{1}{3}$ (B) $x = \frac{2}{3}$ (C) $x = 2$ (D) $x = 4$ (E) NOTA

11. Solve for $x > 0$: $x^{x^{x^{\cdot^{\cdot^{\cdot}}}}} = 3$

    (A) $x = \sqrt[3]{3}$ (B) $x = \sqrt{3}$ (C) $x = 3$ (D) $x = 9$ (E) NOTA

12. Solve for x: $\ln x + \ln 5x = 14 - 3\ln 125$

(A) $\{e^{14}5^{-10}\}$ (B) $\{e^{7}5^{-5}\}$ (C) $\{\pm e^{14}5^{-10}\}$ (D) $\{\pm e^{7}5^{-5}\}$ (E) NOTA

13. If $f(x) = b^x$ and $0 < b < 1$, then the graph of $y = f(x)$ has which of the following?

(A) A horizontal asymptote only

(B) A vertical asymptote only

(C) Both a horizontal and a vertical asymptote

(D) Neither a horizontal nor a vertical asymptote

(E) NOTA

14. Solve for x: $4^{44} + 4^{44} = 4^x$

(A) $x = 44$ (B) $x = \frac{89}{2}$ (C) $x = 88$ (D) $x = 89$ (E) NOTA

15. What is the domain of $f(x) = 5 - \ln(3 - 4x)$?

(A) $(-\infty, \infty)$ (B) $(-\infty, \frac{3}{4})$ (C) $(\frac{3}{4}, \infty)$ (D) $(5, \infty)$ (E) NOTA

16. Find the sum of the solutions for x: $2^{x+3} - 4^x = 12$

(A) 8 (B) $2 - \log_2 3$ (C) $\log_2 6$ (D) $1 + \log_2 6$ (E) NOTA

17. How many of the following are true?

I. $\ln e^a = a$

II. If $\ln(a^2)$ exists, then $\ln(a^2) = 2\ln a$

III. If $\ln(ab)$ exists, then $\ln(ab) = \ln a + \ln b$

IV. If $\sum_{i=1}^{n} \ln x_i$ exists for all x_i and $n > 1$, then $\sum_{i=1}^{n} \ln x_i = \ln\left(\prod_{i=1}^{n} x_i\right)$

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

18. Find the sum of the following terms for $N > 1$: $\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + \ldots + \frac{1}{\log_{25} N}$

(A) $\log_N 324$ (B) $\log_5 N$ (C) $\log_{25} N$ (D) $\log_N 25!$ (E) NOTA

19. Solve for x: $27^{5x} = \tan\frac{4\pi}{3}$

(A) $-\frac{1}{30}$ (B) $-\frac{1}{10}$ (C) $\frac{1}{10}$ (D) $\frac{1}{30}$ (E) NOTA

20. If $\log_{243} 343 = a$, then which of the following is equivalent to $\log_{49}\left(\frac{1}{27}\right)$?

(A) $\frac{3}{2}a$ (B) $-\frac{3}{2a}$ (C) $-\frac{9}{10a}$ (D) $\frac{5}{2}a$ (E) NOTA

21. What is the characteristic of $\log_4 10000$?

(A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA

22. Simplify $\ln\left[\prod_{i=1}^{n}\left(\sum_{j=0}^{\infty} f(i,j)\right)\right]$ if $f(a,b) = \dfrac{a^b}{g(b)}$ and $g(n) = \begin{cases} n \cdot g(n-1) & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$.

(A) $(n-1)!$ (B) $n!$ (C) $\frac{n(n+1)}{2}$ (D) n (E) NOTA

23. Which of the following is equivalent to $(1 - i\sqrt{3})^{-7}$? $(i = \sqrt{-1})$

(A) $-\frac{1}{128} - \frac{\sqrt{3}}{128}i$ (B) $-\frac{1}{128} + \frac{\sqrt{3}}{128}i$ (C) $\frac{1}{128} - \frac{\sqrt{3}}{128}i$ (D) $\frac{1}{128} + \frac{\sqrt{3}}{128}i$ (E) NOTA

24. If $f(x) = e^x + e^{-x}$, then which of the following is equivalent to $f^{-1}(2)$?

(A) $\ln 1$ (B) $\ln 2$ (C) $\ln e$ (D) $\ln e^{-1}$ (E) NOTA

25. If a and b are the lengths of legs of a right triangle and c is the length of the hypotenuse, which of the following is true?

(A) $\ln(a^2) + \ln(b^2) = \ln(c^2)$ (B) $\log_c(a^2 + b^2) = 2$

(C) $\ln(a^2 + b^2 - c^2) = 0$ (D) $2\ln(a + b - c) = 0$

(E) NOTA

26. If $\log_a 2 = x$, $\log_a 3 = y$, and $\log_a 5 = z$, express $\log_a \left(\frac{4500}{a^2}\right)^4$ in terms of x, y, and z.

(A) $4(x + y + z - 2)$ (B) $4(2x + 2y + 3z) - 2$

(C) $4(2x + 2y + 3z - 2)$ (D) $(2x + 2y + 3z - 2)^4$

(E) NOTA

27. Where defined, $\ln \sin x + \ln \cos x =$

(A) $\ln \sin 2x$ (B) $\ln \sin 2x - \ln 2$

(C) $\ln(\sin x + \cos x)$ (D) $\ln \cos 2x$

(E) NOTA

28. If $n > 1$, then $e^{\left(\sum_{i=1}^{n} \ln i\right)} =$

(A) $\ln n$ (B) e^n (C) $e^{n!}$ (D) $\ln(n!)$ (E) NOTA

29. If $f(x) = e^x$, then $f(a + b) =$

(A) $f(a) + f(b)$ (B) $f(ab)$ (C) $f(a)f(b)$ (D) $f(a) - f(b)$ (E) NOTA

30. Let x be a positive integer. Which of the following represents the n^{th} digit of x from the left? Assume that the first digit of x is not 0.

(A) $\left\lfloor x \cdot 10^{n+1-\lfloor \log x \rfloor} \right\rfloor \bmod 10$

(B) $\left\lfloor x \cdot 10^{n-\lfloor \log x \rfloor} \right\rfloor \bmod 10$

(C) $\left\lfloor x \cdot 10^{n-1-\lfloor \log x \rfloor} \right\rfloor \bmod 10$

(D) $\left\lfloor x \cdot 10^{1-n} \right\rfloor \bmod 10$

(E) NOTA