

1) A

Since $r^2 = x^2 + y^2$ and $\sin(\theta) = \frac{x}{r}$, multiplying the equation by r leads to the equation of a CIRCLE.

2) B

$(x - 4)^2 + (y - 3)^2 = 16$ leads to B

3)E

$1(2) + 1(-10) + 1(-1) = -9$ so E.

4) B

Since the two planes are parallel, we can just pick one point on a plane, and then employ the point to a plane formula. We can easily check that $(2,1,-1)$ is on the first plane. Thus,

using that aforementioned formula, $\frac{\|3(2) + 2(1) - 2(-1) - 12\|}{\sqrt{3^2 + 2^2 + (-2)^2}} = \frac{2\sqrt{17}}{17}$.

5) B

This conic section can be written as $\frac{(x - 2)^2}{4} - \frac{(y - 1)^2}{9} = 1$ and thus, the slopes of the asymptotes are $\pm\frac{3}{2}$. So a = $\frac{3}{2}$

6) B

Much like normal trig, hyperbolic functions have the property that $\sinh^2 x - \cosh^2 x = -1$, always. Therefore B.

7) E

We can treat each point as a vector and use the cosine relation to get $\theta = \arccos(\frac{\sqrt{10}}{10})$, so E.

8) C

First we must put the points in order, than we can employ the commonly known trick:

$$\begin{array}{r} |0 \ 1| \\ 2|2 \ 2|0 \\ 6|3 \ 3|6 \\ 9|3 \ 8|24 \\ -8|-1 \ 4|12 \\ 0|0 \ 1|-1 \end{array}$$

$$\begin{aligned} &= |9 - 41|/2 \\ &= 16 \text{ so C} \\ &9) \text{ B} \end{aligned}$$

I) False, since a parabola does not have to be a function, there are an infinite of parabolas that can go through three points (they will have xy terms). II) True. This rational function when graphed is clearly a hyperbola (it has an xy term in standard form).

10) A

One can easily derive the rotation matrix with the use of trig and trig sum formulas. We will use it due to its elegance. So for this problem:

$$\begin{aligned} &\begin{bmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{7\sqrt{3}}{2} - \frac{1}{2} \\ \frac{7}{2} + \frac{\sqrt{3}}{2} \end{bmatrix} \\ &11) \text{ D} \end{aligned}$$

We will rewrite the equation of the line as $y = (3/4)x + 1/4$ and plugging this into the equation of the circle leads to $x^2 - 10x + 9 = 0$. Factoring this equation leads to $(x - 1)(x - 9) = 0$ and thus the x -coordinates are 9 and 1. Plugging these value into the equation of the line give us the y values which are in choice D.

12) A

This function equals $k \sin(4x)$ which has a period of $\pi/2$

13) B

This relation is equal to $x^2/25 + y^2/9 = 1$ which has a max x value of 5 and a max y value of 3. Therefore $ab = 15$ which has 4 divisors.

14) C

$$A^2 = 4^2 + 19^2 = 377$$

15) A

Since the radius is orthogonal to the plane, it is also the shortest distance from the center to the point. So we can use the point to a plane formula

$$\rightarrow \frac{|1(1) - 2(2) - 2(2) - 10|}{\sqrt{1^2 + 2^2 + 2^2}} = 1/3. \text{ So A.}$$

16) B

A median of a triangle divides the triangle into two of equal area, because the height and base of both triangles are equal. The median from A to BC intersects BC at its midpoint: $(15/2, 17/2)$. Therefore, the slope from A to this point is $11/13$

17) E

$f(x)$ is even and therefore the new relation would be equal to the original one.

18) C

$$\text{This equals } (2i)^2 * (1 + i) = -4 - 4i$$

19) D

Taking the cross product of the two vectors results in a vector that is a multiple of D. Therefore D is perpendicular to them both.

20) E

Solve each equation for their respective trig function. Squaring each equation and then adding them yields the equation of a circle which has an eccentricity of 0.

21) B

To make the function continuous, plug 2 for x into both parts and set them equal to each other. This leads to $(k-1)(k-5)(k^2+k+1) = 0$. Since k must be real, the sum is equal to 6.

22) D

Subtracting A from B will result in the direction vector from A to B. Dividing this by 3 and then adding it to A will result in the desired result. D.

23) A

$f(x) = \cosh x$ and $g(x) = 1 + 2 \sinh x$. By hyperbolic identities, $g(x) = f(2x)$. One could just expand the expression and it would be clear that $g(x) = f(2x)$.

24) D

$A = 2, B = 4, C = 2, B^2 - 4AC = 16 - 4 * 2 * 2 = 0$ Therefore the conic section is a parabola and parabolas always have an eccentricity of 1.

25) B

Graphing the two rectangles, we can see the intersection is a rectangle. This rectangle has side lengths (10-6) and (8-6). Therefore, its area is equal to (4)(2) = 8.

26) B

Taking (1,0,5) as an origin, and subtracting from the other two points, we can use a determinant to find the volume: $\begin{vmatrix} 0 & 3 & 12 \\ 0 & 6 & 6 \\ 1 & 2 & 9 \end{vmatrix} = -54$. We must take the absolute value and then divide this by 6 to get the true volume of 9.

27) D

Setting the two equations equal to each other yields $x^2 - 6x + 8$. The product of the solutions is 8.

28) B

We are trying to maximize $\frac{y}{x}$. We can say $\frac{y}{x} = c$ and therefore $y = cx$ where the maximum value for c is what we are looking for. $y = cx$ is a line and $x^2 - 10x + y^2 - 6y + 30 = 0$ is a circle and the intersection of these two graphs (when c has a known value) will give where on the graph the ratio of y and x is c . Since c is nothing but the slope of the line, we are trying to maximize the slope of the line when it intersects the circle, and that happens when the line is tangent to the circle (we must first verify that the circle does not cross the y -axis otherwise the maximum value would approach infinity).

If we plug in for every y on $x^2 - 10x + y^2 - 6y + 30 = 0$, cx , we can find the x -value when c is known, but since we only want there to be one solution of this quadratic, we set the discriminant equal to 0. This will give us the value of the slope of the line when it is

tangent to the circle (occurs in different points, one is when the slope (and consequently $\frac{y}{x}$) is a maximum and the other is when it is a minimum). So: $x^2 - 10y + y^2 - 6y + 30$, $x^2 - 10x + (cx)^2 - 6cx + 30 = 0$, $(1 + c^2)x^2 - (10 + 6c)x + 30 = 0$. The discriminant equals $b^2 - 4ac$, $(-10 - 6c)^2 - 4(1 + c^2)(30) = 0$, $100 + 120c + 36c^2 - 120 - 120c^2 = 0$, $84c^2 - 120c + 20 = 0$.

Using the awesome quadratic formula yields $\frac{5}{7} \pm \frac{2\sqrt{30}}{21}$ and the bigger of the two values will be the maximum so therefore B.

29) C

I) True since an asymptote is just the behavior of a function as a variable goes to infinity, it can cross it some where earlier.

II) This is true by definition

30) E

Making the substitution of $\sin 2x = 2 \cos x \sin x$, we see that $\sin x = .5$, $\cos x = 0$, and $x = 0$. None of the choices cover all cases.