1. (**10 points**) Write a program that takes as input a positive integer $n$, and displays
   (a) the value of $n^2$
   (b) the $n^{\text{th}}$ triangular number $T_n$ ($T_1 = 1$, $T_n = n + T_{n-1}$)

   **Input:**
   ```
   9
   ```

   **Output:**
   ```
   (a) 81
   (b) 45
   ```

2. (**10 points**) Consider the equation $ax + b = c$ with $a \neq 0$. Write a program that takes integers $a$, $b$, and $c$ as input, and displays the solution to this equation for $x$.

   **Input:**
   ```
   a = 3
   b = 16
   c = 22
   ```

   **Output:**
   ```
   x = 2.0
   ```

3. (**10 points**) Write a program that takes as input a string of only letters and displays the string with every third letter capitalized starting with the second letter, and all other letters lower-case.

   **Input:**
   ```
   bulldogsbulldogsbowwowwow
   ```
   **Output:**
   ```
   bUllDogSbuLldOgsBowWowWow
   ```

4. (**10 points**) Write a program that takes as input integers $a$, $b$, $c$, and $d$ and displays the equation of the line (in slope-intercept form, where applicable) between the distinct points in the plane $(a, b)$ and $(c, d)$.

   **Input:**
   ```
   a = 1
   b = 35
   c = -3
   d = -1
   ```

   **Output:**
   ```
   y = 9.0x + 26.0
   ```

5. (**10 points**) Write a program that takes as input a positive integer $n$ no greater than 99999, and displays the value of
   $$\sum_{i=1}^{(\text{Number of digits of } n)} (\text{The } i^{\text{th}} \text{ digit of } n)^i.$$
   (i.e.: For 82736, the output should be $8^1 + 2^2 + 7^3 + 3^4 + 6^5 = 8212$.)

   **Input:**
   ```
   92686
   ```
   **Output:**
   ```
   12101
   ```

6. (**20 points**) Write a program that takes as input two integers $a$ and $b$ with $a < b$, and displays the sum of the squares of the integers between $a$ and $b$ inclusive.

   **Input:**
   ```
   a = 9
   b = 26
   ```

   **Output:**
   ```
   5997
   ```

7. (**20 points**) Write a program that takes as input a string, and displays whether or not that string is a palindrome when spaces and capitalization are ignored. (A palindrome is a string which is the same when read forwards or backwards.)

   **Input:** `Able was I ere I saw elba`
   **Output:** `YES`

   **Input:** `Handsome Dan`
   **Output:** `NO`

8. (**20 points**) Consider the triangle with two sides of length $a$ and $b$ and an angle $C$ between them. Write a program that takes as input two positive numbers, $a$ and $b$, and an angle $C$ expressed in degrees ($0° < C < 180°$), and displays

   (a) the area of the triangle

   (b) the length of the third side of the triangle

   **Input:**
   ```
   a = 9
   b = 26
   C = 86
   ```

   **Output:**
   ```
   (a) 116.71499
   (b) 26.91382
   ```

9. (**20 points**) Write a program that takes as input a string $s$ and a character $c$, and displays how many times and at what character positions $c$ occurs within $s$. (Consider the first character as occupying position 1.)

   **Input:**
   ```
   s:  How much wood would a woodchuck chuck if a woodchuck could
       chuck wood
   c:  w
   ```

   **Output:** `6 occurences at positions 3 10 15 23 44 66`

10. (**20 points**) Write a program that takes as input two five-digit positive integers $a$ and $b$ and a positive integer $c$, and displays the sum of all integers between $a$ and $b$ (inclusive) whose digits sum to $c$. (i.e.: if, for example, $x$ and $y$ are the only two integers in $[a, b]$ whose digits sum to $c$, then the program should display the value of $x + y$.)

    **Input:**
    ```
    a = 27182
    b = 31415
    c = 18
    ```

    **Output:** `6357843`

11. (**30 points**) Write a program that takes as input a word (a string of only letters), and displays the Bordoley value of that word. The Bordoley value of a string is obtained by subtracting the sum of the values of the letters in even-numbered positions in the string from the sum of the values of the letters in odd-numbered positions in the string. The value of a letter is given by its position in the alphabet, i.e. the value of the letter `A` is 1, that of the letter `B` is 2, etc. (For example, the Bordoley value of the word `Limor` is $12 - 9 + 13 - 15 + 18 = 19$.)

**Input:** `William`
**Output:** `35`

**Input:** `Meow`
**Output:** `0`

12. (**30 points**) Consider the convex quadrilateral with vertices at $(0, 0)$, $(0, a)$, $(b, c)$, and $(d, 0)$ with $a$, $b$, $c$, and $d$ positive integers. Write a program that takes $a$, $b$, $c$, and $d$ as input, and displays

    (a) whether or not the quadrilateral is a square (`YES` or `NO`)

    (b) the area of the quadrilateral

    (c) the equations, in slope-intercept form, of the lines containing the diagonals of the quadrilateral

    (d) the point of intersection of the diagonals of the quadrilateral

**Input:**
```
a = 1
b = 2
c = 3
d = 4
```

**Output:**
```
(a) NO
(b) 7.0
(c) y = 0.25x + 1.0 , y = 1.5x
(d) (0.57143, 0.85714)
```

13. (**30 points**) Write a program that takes as input a positive integer, and displays the Bantz of that integer. The Bantz of a positive integer $a = a_1 a_2 \dots a_n$, where $n$ is the number of digits in the number and $a_1 \dots a_n$ are the digits ($a_k$ is the $k^{\text{th}}$ digit of $a$), is given by the recursive definition

$$B(a_1 a_2 \dots a_n) = \begin{cases} F_{(a_1+1)}(a_1^n + B(a_2 \dots a_n)) & \text{if } n > 1 \\ a_1! & \text{if } n = 1, \end{cases}$$

where $F_k$ is the $k^{\text{th}}$ Fibonacci number (given by $F_1 = F_2 = 1$, $F_n = F_{n-1} + F_{n-2}$). Thus, $F_{(a_1+1)}$ refers to the $(a_1 + 1)^{\text{th}}$ Fibonacci number, where $a_1$ is the first digit of $a$.
For example, applying this definition to compute $B(31415)$ gives

$$\begin{aligned} B(31415) &= F_4(3^5 + B(1415)) \\ &= 3(243 + B(1415) \\ &= 3(243 + F_2(1^4 + B(415))) \\ &\dots \end{aligned}$$

**Input:** `31415`
**Output:** `3507`

14. (**30 points**) Write a program that takes a message from planet Singleton and decodes it into English. A message is a string of letters and spaces. Messages from planet Singleton are decoded according to the following rules:

    (a) Each word in the message is decoded as a character, and spaces in the message do not have any decoded values.

    (b) If a word in the message has two or fewer letters, its decoded version is a space.

    (c) If a word in the message has three or five letters, its decoded version is the last letter of the word.

    (d) If a word in the message has four letters, its decoded version is the letter whose value is equivalent to the sum of the values of the letters in the word modulo 26. The value of a letter is given by its position in the alphabet (the value of `A` is 1, that of `B` is 2, etc.) Note that `Z` is equivalent to 0 `mod` 26. Thus, for example, the decoded version of `SENN` is `Z`, since sum of the values of the letters in `SENN` is $19 + 5 + 14 + 14 = 52 \equiv 0 \bmod 26$.

    (e) A word in the message with six or more letters indicates the message is over, and is decoded as a period. Any subsequent words in the message should be ignored.

    **Input:** `Red goo must hug true alpha fund at low chipi fone snow matrix`
    **Output:** `DOUGLAS WINS.`

15. (**30 points**) The word *abstemious* possesses a unique property: each vowel of the alphabet ($a$, $e$, $i$, $o$, $u$) appears exactly once, and the vowels are in alphabetical order within the word. Write a program that takes as input a string of only letters and determines whether the string possesses this property.

    **Input:** `aabstemious`
    **Output:** `NO`

    **Input:** `aBCdEFgHIjKLmNOpQRsTUvWXyZ`
    **Output:** `YES`

FAMAT 2006 State Convention

# Computer Programming Competition

- This one-hour computer programming competition will consist of 15 problems, each of which will require students to write a program that displays correct output for each input value or set of input values provided by the judge. There are 10-point, 20-point, and 30-point problems, which may be solved in any order.
- Each school is limited to one team of three members and must supply its own computer and necessary power cables. Acceptable programming languages are BASIC, Pascal, C/C++, and Java. No calculators or cell phones are permitted in the room. One programming language reference book is allowed.
- If a program does not display the correct output for the input value(s) supplied by the judge on the team's first attempt, the team will receive one more opportunity to have the problem judged, but *a correct second attempt will be worth only half as many points as a correct first attempt.*
- Problems with multiple parts must be solved completely to receive credit. *If any part of the output is incorrect, no points will be awarded for that attempt.*
- If you wish to have a problem graded, one member of your team should raise his/her hand and wait for a judge to come to your area. The judge will provide input and will check to make sure your program displays the correct output for the value(s) provided.
- For each problem, the judge may ask you to run your program one, two, or three times with different inputs. *If your program fails to display the correct output during any of these runs, no points will be awarded for that attempt.*
- Whenever you ask to have a problem judged, the judge must initial your score sheet in the appropriate location to indicate whether or not the problem was solved correctly and whether this is your first or second attempt.
- Unless otherwise indicated, numerical input and output values will be real numbers, and string input values will not exceed 100 characters in length.
- Round-off error in output values will be ignored, provided the output is sufficiently accurate to determine that it is the correct answer. For example, the value `0.57143` is acceptable for $\frac{4}{7}$, but `0.6` is not.
- The sample input and output accompanying each problem do not necessarily test all possibilities.