

1. **(A)** $x + 1 > 2 \Rightarrow x + 1 - 1 > 2 - 1 \Rightarrow x > 1$
2. **(C)** $x = y + 1 = (z - 1) + 1 = z = 1$
3. **(B)** $3x = 9 \Rightarrow x = \frac{1}{3}(9) = 3 \Rightarrow 5x + 11 = 5(3) + 11 = 26$
4. **(D)** $3^{4x} = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3} = 3^{-\frac{1}{2}} \Rightarrow 4x = -\frac{1}{2} \Rightarrow x = -\frac{1}{8}$
5. **(A)** $3x + 4 = 22 \Rightarrow 3x = 18 \Rightarrow x = 6$ and $6 - y = 12 \Rightarrow -y = 6 \Rightarrow y = -6$, so $x + y = 6 + (-6) = 0$.
6. **(C)** $x^2 = i = \text{cis}(90^\circ) \Rightarrow x = [\text{cis}(90^\circ)]^{\frac{1}{2}} \Rightarrow x = \text{cis}(45^\circ) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ or $x = \text{cis}(225^\circ) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$
7. **(E)** $\text{Sin}^{-1} \left(\sin \left(\frac{7\pi}{6} \right) \right) = \text{Sin}^{-1} \left(-\frac{1}{2} \right) = -\frac{\pi}{6}$
8. **(D)** $x - 2y = 1 \Rightarrow x = 2y + 1 \Rightarrow x + y = (2y + 1) + y = 3y + 1 = 16 \Rightarrow 3y = 15 \Rightarrow y = 5 \Rightarrow x = 2(5) + 1 = 11$, so $5x - 11y = 5(11) - 11(5) = 0 \neq 30$. Choices (A), (B), and (C) are all true.
9. **(E)** The sum of the 7th roots of 2 is the sum of the solutions to $x^7 - 2 = 0$. The sum of the roots of any polynomial equation $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 = 0$ is $-\frac{a_{n-1}}{a_n}$. The coefficient of x^6 is 0, so the sum is 0.
10. **(B)** $x^2 - 6x + 5 = (x - 1)(x - 5) > 0 \Rightarrow x < 1$ or $x > 5$
 - I. $-|x - 3| < -2 \Rightarrow |x - 3| > 2 \Rightarrow x - 3 < -2$ or $x - 3 > 2 \Rightarrow x < 1$ or $x > 5$ (equivalent)
 - II. Discriminant $= (-6)^2 - 4(1)(-5) = 56 \Rightarrow$ irrational roots \Rightarrow not equivalent to $x^2 - 6x + 5 > 0$
 - III. $1 < x < 5$ and $x^2 - 6x + 5 > 0$ are mutually exclusive, so they are not equivalent.
 - IV. $\frac{1}{10}|(3x - 9) + (4x - 12)i| = \frac{1}{10}\sqrt{(3x - 9)^2 + (4x - 12)^2} = \frac{1}{10}\sqrt{25x^2 - 150x + 225} = \frac{1}{2}\sqrt{(x - 3)^2} = \frac{1}{2}|x - 3| > 1 \Rightarrow |x - 3| > 2 \Rightarrow$ equivalent to I.
11. **(A)** $\frac{1}{\ln 2}[1 - S(\text{Arcsin } x)] = 2S(e) \Rightarrow S(\text{Arcsin } x) = 1 - 2 \cdot \frac{\ln e}{\ln \pi} \cdot \ln 2 = \log_\pi \pi - 2 \log_\pi 2 = \log_\pi \left(\frac{\pi}{4} \right)$
 $S(\text{Arcsin } x) = \log_\pi(\text{Arcsin } x) = \log_\pi \left(\frac{\pi}{4} \right) \Rightarrow \text{Arcsin } x = \frac{\pi}{4} \Rightarrow x = \frac{\sqrt{2}}{2} = .7071\dots$
12. **(D)** $f(\sin \theta) = 1 - 2 \sin^2 \theta = \cos 2\theta$
13. **(A)** $2^{6x} + 4^{9+3x} = 2^{6x} + 2^{6x+18} = 2^{6x}(1 + 2^{18}) = 4 + 2^{20} \Rightarrow 2^{6x} = \frac{4 + 2^{20}}{1 + 2^{18}} = \frac{4(1 + 2^{18})}{1 + 2^{18}} = 4 = 2^2 \Rightarrow 6x = 2 \Rightarrow x = \frac{1}{3}$

14. **(B)** $A^{-1} = \frac{1}{\det A} \begin{bmatrix} 1 & -4 \\ -3 & 11 \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 1 & -4 \\ -3 & 11 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 3 & -11 \end{bmatrix}$
 $AX = B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow (A^{-1}A)X = IX = X = A^{-1}B$
 $X = A^{-1}B = \begin{bmatrix} -1 & 4 \\ 3 & -11 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} (-1)(2) + (4)(1) & (-1)(7) + (4)(8) \\ (3)(2) + (-11)(1) & (3)(7) + (-11)(8) \end{bmatrix} = \begin{bmatrix} 2 & 25 \\ -5 & -67 \end{bmatrix}$
 $2 + 25 + (-5) + (-67) = -45$
15. **(B)** $[x]$ represents the integer part of x , which is equivalent to x only if x is an integer.
16. **(A)** $x_1 + x_2$ is simply the sum of the solutions to the equation $f(x) = 1 - x^2 = a$. This equation is the same as $x^2 + (a - 1) = 0$, and it is readily apparent that the sum of the solutions to this equation is 0. Thus, $(x_1 + x_2)^2 = 0^2 = 0$.
17. **(B)** $\ln x + \ln(3x) = \ln 3x^2 = 12 + \ln 3 = \ln 3e^{12} \Rightarrow x^2 = e^{12} \Rightarrow x = \pm e^6$. $x = -e^6$ is extraneous because $\ln x$ is not defined for $x = -e^6$.
18. **(D)** $y = x \sin \theta + \cos \theta \Rightarrow x = \frac{y - \cos \theta}{\sin \theta} = y \csc \theta - \cot \theta$
19. **(B)** If $\vec{u} = \langle u_1, u_2 \rangle$, then $\vec{u} \bullet \langle 9, 26 \rangle = 9u_1 + 26u_2 = 0 \Rightarrow u_2 = -\frac{9}{26}u_1$, so II. is false. It can be shown using the law of cosines that the cosine of the angle θ between two vectors \vec{u} and \vec{v} is $\cos \theta = \frac{\vec{u} \bullet \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$, which implies that the dot product of two vectors is 0 if and only if they are orthogonal, i.e. they make an angle of 90° . Thus, I. and III. are false, and IV. is true.
20. **(A)** $2x^2 \cot \theta + \sin 2\theta = 4x \Rightarrow 2x^2 - 4x + 2 \sin \theta \cos \theta = 0$
 $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2 \cot \theta)(2 \sin \theta \cos \theta)}}{2(2 \cot \theta)} = \frac{4 \pm 4\sqrt{1 - \cos^2 \theta}}{4 \cot \theta} = \tan \theta(1 \pm \sin \theta) =$
 $\frac{\sin \theta \pm \sin^2 \theta}{\cos \theta} = \frac{\sin \theta \pm (1 - \cos^2 \theta)}{\cos \theta} = \tan \theta \pm \sec \theta \mp \cos \theta$
21. **(A)** $x^2 + 2ax = 3a \Rightarrow x^2 + 2ax - 3a = 0$
Determinant = $(2a)^2 - 4(1)(-3a) = 4a^2 + 12a = 4a(a + 3) < 0 \Rightarrow -3 < a < 0$
22. **(C)** The region is outside the circle $x^2 + y^2 = 9$ and inside the rhombus with vertices $(-5, 0)$, $(5, 0)$, $(0, 10)$, and $(0, -10)$. The circle lies completely within the rhombus. The area of the circle is $\pi r^2 = 9\pi$, and the area of the rhombus is $\frac{1}{2}$ the product of the diagonal lengths, or $\frac{1}{2}(10)(20) = 100$. Thus, the area of the region is $A = 100 - 9\pi$. $\cos A = \cos(100 - 9\pi) = -.8623\dots$

23. **(C)** $2 + 4 + 6 + \dots + 2n$ is an arithmetic series with n terms, first term 2, and last term $2n$, so its value is $\frac{1}{2}n(2 + 2n) = n(n + 1) \neq n^2 + 1$. (A) and (B) can be confirmed similarly, and (D) can be proved easily by induction.

24. **(B)** $1 + \sin^2 x + \sin^4 x + \sin^6 x + \dots = \sum_{i=0}^{\infty} (\sin^2 x)^i = \frac{1}{1 - \sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x$

The two sides of the equation are equivalent for all x for which $\sec x$ and $\tan x$ are defined (all x except $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$). Thus, the solution set is $\{x \mid x \neq \frac{n\pi}{2}\}$, where n is an odd integer.

25. **(D)** Since $0 < a < c$, $a - c < 0$. Then $ax + b < cx + d \Rightarrow (a - c)x < d - b \Rightarrow x > \frac{d - b}{a - c}$

26. **(B)** The solution set of $\sin(\pi x) = 0$ is $\{0, \pm 1, \pm 2, \pm 3 \dots\}$.

I. Solution set: $\{0, \pm 1, \pm 2, \pm 3 \dots\}$

II. Solution set: $\{0, 1, 2, 3 \dots\}$

III. Solution set: $\{0, \pm 2, \pm 4, \pm 6 \dots\}$

IV. Solution set: $\{0, \pm 1, \pm 2, \pm 3 \dots\}$

2 of the equations have the solution set $\{0, \pm 1, \pm 2, \pm 3 \dots\}$.

27. **(A)** To reflect a curve about the y -axis, replace x with $-x$. Thus, when the curve is reflected about the y -axis, the equation becomes $y = (-x)^2 - 6(-x) + 16 = x^2 + 6x + 16$.

28. **(E)** $\{x \mid \sqrt{(x - \pi)^2} \leq \pi\} = \{x \mid |x - \pi| \leq \pi\} = \{x \mid 0 \leq x \leq 2\pi\} = [0, 2\pi]$

$$\{x \mid \sec x \leq 2\} = \{x \mid \cos x \geq \frac{1}{2}\} \cup \{x \mid \cos x < 0\} = ([0, \frac{\pi}{3}] \cup [\frac{5\pi}{3}, 2\pi]) \cup (\frac{\pi}{2}, \frac{3\pi}{2}) = [0, \frac{\pi}{3}] \cup (\frac{\pi}{2}, \frac{3\pi}{2}) \cup [\frac{5\pi}{3}, 2\pi]$$

$$\{x \mid \csc x \leq 2\} = \{x \mid \sin x \geq \frac{1}{2}\} \cup \{x \mid \sin x < 0\} = [\frac{\pi}{6}, \frac{5\pi}{6}] \cup (\pi, 2\pi)$$

$$\{x \mid \cot x > 0\} = \{x \mid \tan x > 0\} = (0, \frac{\pi}{2}) \cup (\pi, \frac{3\pi}{2})$$

The intersection of all four sets is $[\frac{\pi}{6}, \frac{\pi}{3}] \cup (\pi, \frac{3\pi}{2})$.

29. **(D)** The determinant is equivalent to

$$x \begin{vmatrix} 7x & -1 \\ 9 & 4-x \end{vmatrix} - (-2) \begin{vmatrix} 3 & -1 \\ 2x+5 & 4-x \end{vmatrix} + (1-x) \begin{vmatrix} 3 & 7x \\ 2x+5 & 9 \end{vmatrix} =$$

$$x[(7x)(4-x) - (-1)(9)] - (-2)[(3)(4-x) - (-1)(2x+5)] + (1+x)[(3)(9) - (7x)(2x+5)] =$$

$$x(48x - 7x^2 + 9) + 2(12 - x + 5) + (1+x)(27 - 14x^2 - 35x) = -21x^3 - 21x^2 - x + 61.$$

For $ax^3 + bx^2 + cx + d = 0$, the sum of the roots (i.e., $x_1 + x_2 + x_3$) is $-\frac{b}{a}$, and the sum of the roots taken two at a time (i.e., $x_1x_2 + x_2x_3 + x_1x_3$) is $\frac{c}{a}$. Thus, the sum of the squares of the roots (i.e., $x_1^2 + x_2^2 + x_3^2$) is given by $(\frac{-b}{a})^2 - 2(\frac{c}{a})$. For this equation, $(\frac{-b}{a})^2 - 2(\frac{c}{a}) = (\frac{21}{-21})^2 - 2(\frac{-1}{-21}) = \frac{19}{21}$. $\sqrt{(19)(21)} = 19.9749\dots$

30. **(C)** For any real number n , $n - [n]$ denotes the fractional part of n and is 0 for any integer. Thus,

$\lfloor \frac{6}{\pi} \arccos \sqrt{x} \rfloor - \frac{6}{\pi} \arccos \sqrt{x} = 0$ implies that $\frac{6}{\pi} \arccos \sqrt{x}$ is an integer, so $\arccos \sqrt{x}$ must be a multiple of $\frac{\pi}{6}$. The values of x for which $\arccos \sqrt{x}$ is a multiple of $\frac{\pi}{6}$ are $0, \frac{1}{4}, \frac{3}{4},$ and 1 .

$$16x(x - \frac{1}{4})(x - \frac{3}{4})(x - 1) = 16x^4 - 32x^3 + 19x^2 - 3x = 0 \Rightarrow a = -32, b = -19, c = -3, d = -1$$

$$(a - b - c)(b + c - d) = (-32 - (-19) - (-3))(-19 + (-3) - (-1)) = 210$$