

**TRIANGLES THETA**  
**FAMAT State Convention 2004**

For all questions, E. NOTA means none of the above answers is correct.

1. Find an expression for the area of a regular triangle given side length  $s$ .

A.  $\frac{s^2}{4}$       B.  $\frac{s^2}{2}$       C.  $\frac{s^2\sqrt{3}}{4}$       D.  $\frac{s^2\sqrt{3}}{2}$       E. NOTA

2. Find the radius of a circle circumscribed about a 5-12-13 (side lengths) triangle.

A. 6      B. 6.5      C. 9      D. 10      E. NOTA

3. A circle of area  $16\pi$  is inscribed within a triangle of area 12. Find the perimeter of the triangle.

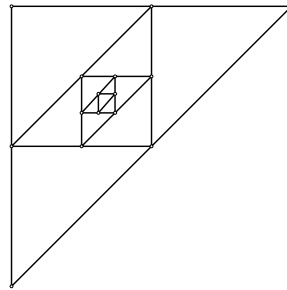
A. 6      B. 3      C.  $\frac{1}{6}$       D.  $\frac{1}{3}$       E. NOTA

4. How long is the shadow of a 3' tall man if it is cast by a 21' tall lamppost 20' away?

A.  $\frac{63}{17}$ ,      B.  $\frac{20}{7}$ ,      C.  $\frac{10}{3}$ ,      D.  $\frac{70}{3}$ ,      E. NOTA

5. A  $45^\circ 45^\circ 90^\circ$  triangle is drawn with leg length 1. Another similar triangle is then drawn inside of it by connecting the midpoints of the sides of the first triangle. This process is carried out an infinite number of times. Find the sum of the perimeters of all triangles drawn. (Do not double count any line segments.)

- A. 4      B.  $2 + \sqrt{2}$  length  
C.  $4 + 2\sqrt{2}$       D. Infinite  
E. NOTA



6. A regular tetrahedron (triangular pyramid) is sliced in half so that the 2 halves are congruent and the cross section of the slice is a triangle. What best describes this cross section?

- A. Equilateral      B. Isosceles      C. Scalene      D. Trapezoidal      E. NOTA

7. How many distinct right triangles with integer side lengths have one side of length 5?

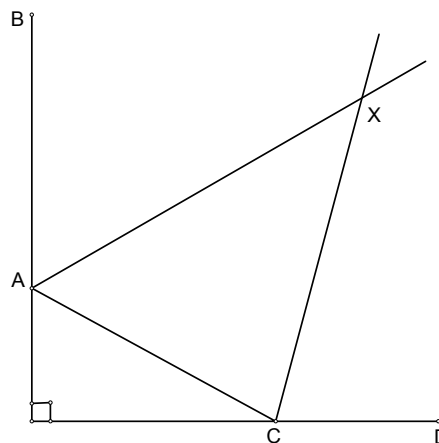
- A. 1      B. 2      C. 3      D. Infinitely many      E. NOTA

8. Find the center of gravity of a triangular region with endpoints  $(-5, 0)$ ,  $(4, 1)$ , and  $(7, 2)$ .
- A.  $(0, 0)$     B.  $(\sqrt[3]{140}, 0)$     C.  $(\frac{140}{9}, 2)$     D.  $(2, 1)$     E. NOTA
9. Which of the following is always equal to the *incenter* of a triangle?
- I. The center of a circle circumscribed about the triangle.  
 II. The center of a circle inscribed within the triangle.  
 III. The intersection of the angle bisectors of the triangle.  
 IV. The intersection of the perpendicular bisectors of the triangle.  
 V. The center of an equilateral triangle.  
 VI. The center of gravity of a triangular region.
- A. I, IV, V only                      B. I, III, VI only                      C. II, IV, V only  
 D. II, III, V only                      E. NOTA
10. A helicopter hovers 10m above the center of an equilateral triangle-shaped hospital roof of side length 30m. How far is the helicopter from any of the 3 signal lights on the corners of the roof?
- A. 20m    B.  $5\sqrt{7}$  m    C.  $\frac{5\sqrt{43}}{2}$  m    D.  $5\sqrt{31}$  m    E. NOTA
11. Syki has a geometry assignment where she must draw triangles that have the following properties: right, acute, obtuse, isosceles, equilateral, and scalene. If she completes the assignment using the fewest possible triangles, how many triangles has she drawn?
- A. 2                      B. 3                      C. 4                      D. 5                      E. NOTA
12. Given:  $\cos(A) = \frac{x}{\sqrt{x^2 + 9y^2}}$ . Find  $\tan(A)$  for  $xy \neq 0$ .

- A.  $\frac{x}{3y}$     B.  $\frac{3y}{x}$     C.  $\frac{x}{\sqrt{2x^2 + 9y^2}}$     D.  $\frac{\sqrt{2x^2 + 9y^2}}{x}$     E. NOTA

13.  $m\angle BAX = m\angle CAX$ .  $m\angle ACX = m\angle DCX$ . Find  $m\angle AXC$ .

- A.  $45^\circ$   
 B.  $60^\circ$   
 C.  $75^\circ$   
 D. Not enough information  
 E. NOTA



14. If a flat circular pizza is cut into the maximum number of pieces using only 4 cuts, how many of the pieces will be triangles? (This pizza is infinitely thin and cannot be folded.)

- A. 0      B. 2      C. 3      D. 5      E. NOTA

15. The angle of elevation from the top of a building to the top of a second building a mile away is  $45^\circ$ . The angle of depression from the top of the first building down to the base of the second building is  $30^\circ$ . How tall is the second building, in miles?

- A.  $\sqrt{3}$       B.  $1 + \sqrt{3}$       C.  $\frac{\sqrt{3}}{3}$       D.  $1 + \frac{\sqrt{3}}{3}$       E. NOTA

16. If a leg of a triangle is equal to the diameter of a circle circumscribed about the triangle, then the triangle must be:

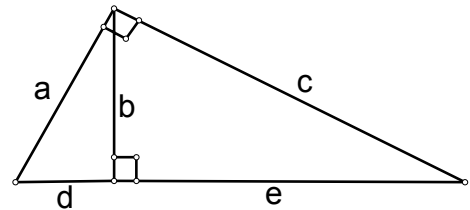
- A. Right      B. Isosceles      C. Obtuse      D. Acute      E. NOTA

17. Find the area of a triangle with vertices (1, 1), (2, 3), (5, 8).

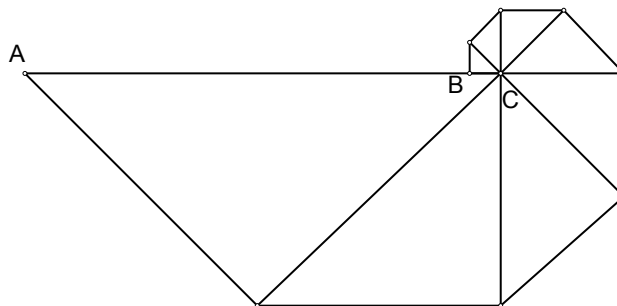
- A. 1      B.  $\frac{1}{2}$       C.  $-\frac{1}{2}$       D. 2      E. NOTA

18. In the figure, what is the geometric mean of  $e$  and  $d+e$ ?

- A.  $a$       B.  $b$       C.  $c$   
 D.  $\frac{b}{c}$       E. NOTA



19. If  $AB = 30$ , what is  $BC$ ? (All triangles are isosceles right triangles.)

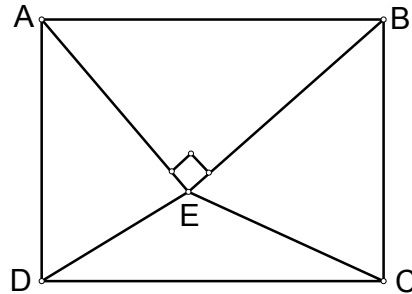


- A. 2      B.  $\sqrt{2}$       C.  $2\sqrt{2}$       D.  $\frac{\sqrt{2}}{2}$       E. NOTA

20. The shortest side of a  $45^\circ 60^\circ 75^\circ$  triangle is 6. Find the length of the second shortest side.
- A.  $2\sqrt{2}$       B.  $3\sqrt{3}$       C.  $3\sqrt{6}$       D.  $3+3\sqrt{3}$       E. NOTA

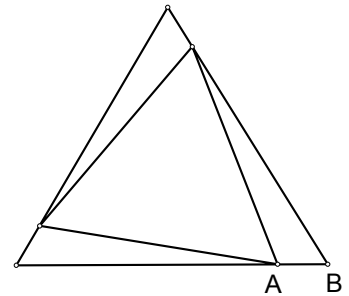
21.  $AE = 6$ ,  $BE = 8$ , and the area of triangle  $BCE = 16$ . Find the area of rectangle  $ABCD$ .

- A. 50      B.  $\frac{800}{9}$   
 C. 32      D. 25  
 E. NOTA



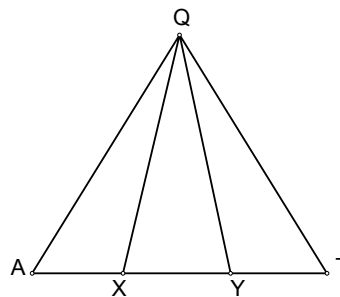
22. An equilateral triangle of area  $\frac{3\sqrt{3}}{4}$  is inscribed in another equilateral triangle as shown in the figure. If  $AB = 1$ , what is the area of the larger equilateral triangle?

- A.  $\frac{3\sqrt{3}}{2}$       B.  $\frac{9\sqrt{3}}{4}$   
 C.  $3\sqrt{3}$       D.  $\frac{3}{2}$       E. NOTA



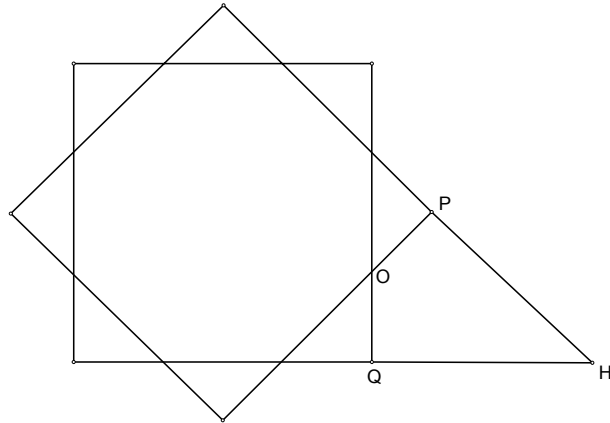
23. Equilateral triangle  $QAT$  is divided into 3 smaller non-overlapping triangles of equal area;  $QAX$ ,  $QXY$ , and  $QYT$ . Find  $XY$  if  $QT = 12$ .

- A.  $\sqrt{3}$       B.  $2\sqrt{3}$   
 C.  $3\sqrt{3}$       D. 4  
 E. NOTA



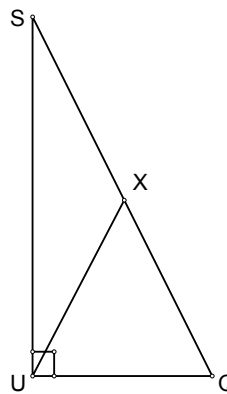
24. The perimeter of the regular octagon in the figure is 16.  
Find the area of quadrilateral QOPH.

- A.  $2 + 2\sqrt{2}$       B.  $1 + 2\sqrt{2}$   
 C.  $6 + 4\sqrt{2}$       D.  $3 + 2\sqrt{2}$   
 E. NOTA



25. In the figure,  $SU = 5$  and  $UQ = 4$ .  $SX = QX$ .  
Find  $UX$ .

- A. 3      B. 4  
 C.  $2\sqrt{5}$       D.  $\frac{\sqrt{41}}{2}$       E. NOTA



26. A and B are right triangles. The short leg of A and B are the same, but the long leg of B is 3 times as long as the long leg of A. What is the ratio of the area of A to the area of B?

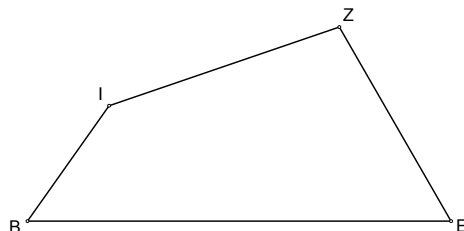
- A.  $\frac{1}{9}$       B.  $\frac{1}{3}$       C.  $\frac{1}{\sqrt{3}}$       D.  $\frac{1}{3\sqrt{3}}$       E. NOTA

27. Right triangle ABC has legs  $AB = \sqrt{33}$  and  $CB = \sqrt{3}$ . Find  $(\sin A)^2 + (\cos A)^2$ .

- A.  $\frac{\sqrt{3} + \sqrt{33}}{6}$       B. 1      C.  $\frac{\sqrt{3} + \sqrt{33}}{36}$       D. 6      E. NOTA

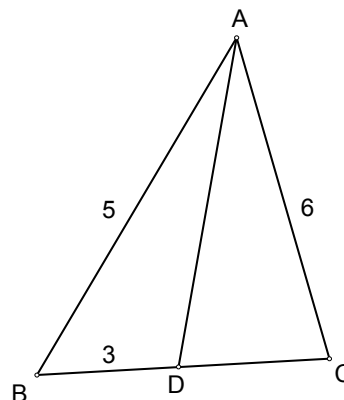
28. In the figure,  $BI = 1$ ,  $BE = 3$ , and  $EZ = 2$ .  $m\angle B = m\angle E = 60^\circ$ . Find  $IZ$ .

- A.  $\frac{\sqrt{13}}{2}$       B.  $\frac{\sqrt{3}}{2}$       C.  $\sqrt{5}$   
 D.  $\sqrt{3}$       E. NOTA



29. If  $DC = \frac{18}{5}$ , what is the ratio of  $m\angle BAD$  to  $m\angle CAD$ ?

- A.  $\sqrt{3} : 2$       B.  $2\sqrt{3} : 3$       C.  $5 : 6$   
D.  $6 : 5$       E. NOTA



30. Two sides of a triangle measure 7 and 12. The third side could be:

- A. 4      B. 14      C. 24      D. All of the above      E. NOTA

## Answers

1. C
2. B
3. A
4. C
5. C
6. B
7. B
8. D
9. D
10. A
11. B
12. B
13. A
14. B
15. D
16. A
17. B
18. C
19. A
20. C
21. A
22. B
23. D
24. A
25. D
26. B
27. B
28. D
29. E
30. B