2003 FAMAT STATE CONVENTION
THETA TEAM SOLUTIONS

① $F(1) = 2(1)+1 = 3$

$G(2) = 2^3 - 2 = 6$

$F(G(3)) = F(25) = 51$

$F^{-1}(4) \Rightarrow 2x+1 = 4$

$x = \frac{3}{2}$

$3+6+51+\frac{3}{2} =$

$\boxed{61.5 \text{ or } \frac{123}{2}}$

② 3 cases:

hyp. x: $\sqrt{5x}^2 + \sqrt{6}^2 = x^2$

$5x+6 = x^2$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x = 6, \cancel{-1}$

hyp. $\sqrt{5x}$: $x^2 + \sqrt{6}^2 = \sqrt{5x}^2$

$x^2+6 = 5x$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$x = 2, 3$

hyp. $\sqrt{6}$: $x^2 + \sqrt{5x}^2 = \sqrt{6}^2$

$x^2+5x = 6$

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$x = \cancel{-6}, 1$

$6+2+3+1 = \boxed{12}$

③ $(1+2)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2}(1)^{\frac{-1}{2}}(2)^1 + \left(\frac{-1}{8}\right)(1)^{\frac{-3}{2}}(2)^2 + \frac{1}{16}(1)^{\frac{-5}{2}}(2)^3 + \left(\frac{-5}{128}\right)(1)^{\frac{-7}{2}}(2)^4 + \frac{7}{256}(1)^{\frac{-9}{2}}(2)^5$

$= 1 + 1 - \frac{1}{2} + \frac{1}{2} - \frac{5}{8} + \frac{7}{8} = \boxed{\frac{9}{4} \text{ or } 2.25}$

④ $A = \frac{72}{32} = \frac{9}{4}$ $\quad B = \frac{1575}{32}$ $\quad C = \frac{-730}{1575} = \frac{-146}{315}$

Sum of squares = (sum of roots)^2 − 2(product taken 2 at a time)

$D = \left(\frac{9}{4}\right)^2 - 2\left(\frac{-564}{32}\right) = \frac{645}{16}$

$\frac{-ACD}{B} = \frac{-\left(\frac{9}{4}\right)\left(\frac{-146}{315}\right)\left(\frac{645}{16}\right)}{\frac{1575}{32}} = \boxed{\frac{3139}{3675}}$

⑤ $A = \sqrt{552+A}$

$A^2 = 552+A$

$A^2 - A - 552 = 0$

$(A-24)(\cancel{A+23}) = 0$

$A = 24$

$B = 1^3 + 2^3 + \ldots + 15^3$

$= (1+2+3+\ldots+15)^2$

$= 120^2 = 14400$

$\frac{2BC}{A} = \frac{2(14400)\left(\frac{5}{16}\right)}{24} = \boxed{375}$

$C = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \ldots = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{4}$

$+ \frac{1}{25} + \frac{1}{125} + \ldots = \frac{\frac{1}{25}}{1-\frac{1}{5}} = \frac{1}{20}$

$+ \frac{1}{125} + \ldots$

Sums are infinite geometric series with same ratio $\frac{1}{5}$ $= \frac{\frac{1}{4}}{1-\frac{1}{5}} = \frac{\frac{1}{4}}{\frac{4}{5}} = \frac{5}{16}$

⑥ $y = 3x+b$

$y = x^2-4x+6$

$3x+b = x^2-4x+6$ has 1 solution $\Rightarrow$ discriminant $b^2-4ac = 0$

$x^2-7x+(6-b)=0$

$\Rightarrow 49-4(1)(6-b)=0$

$49-24+4b=0$

$b = \boxed{\frac{-25}{4} \text{ or } -6.25}$

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(7) $420 + 2\left(\frac{5}{6}\right)(420) + 2\left(\frac{5}{6}\right)^2(420) = 1703.\overline{3}$ before losing bounciness

last height $= \left(\frac{5}{6}\right)^2(420) = 291.\overline{6} = \frac{875}{3}$

after bounciness $= 2\left(\frac{875}{6} + \frac{875}{12} + \frac{875}{24} + \cdots\right) = 2\left(\frac{\frac{875}{6}}{1-\frac{1}{2}}\right) = \frac{1750}{3}$

total $= 1703.\overline{3} + \frac{1750}{3} = \boxed{\frac{6860}{3} \text{ or } 2286.\overline{6}}$

(8) $\frac{16.5 \text{ gal acid}}{100 \text{ gal overall}}$ $\qquad \frac{16.5+x}{100+x} < 0.4$

$16.5 + x < 40 + .4x$

$x < 39.1\overline{6}$ gal of acid

$\frac{39.1\overline{6}}{0.12} = 326.3\overline{8}$ $\qquad$ 326 whole containers

$\boxed{326}$

(9) $A = [\log 4^{44}] + 1 = [256 \log 4] + 1 = 155$

$B \Rightarrow 7^1 = 7,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401$ (repeat) $\quad 8^1 = 8,\ 8^2 = 64,\ 8^3 = 512,\ 8^4 = 4096$ (repeat) $\quad 9^1 = 9,\ 9^2 = 81$ (repeat)

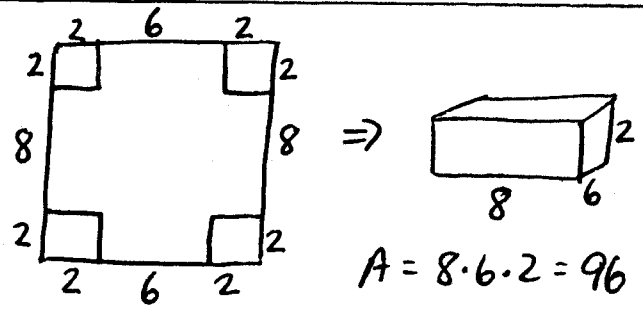
7^{2003} ends in 3 $\quad 8^{2003}$ ends in 2 $\quad 9^{2003}$ ends in 9

$3 + 2 + 9 = 14$ ends in 4 $\quad B = 4$

$C \Rightarrow$ second to last digits' pattern: 1, 2, 3, ..., 0 then repeat

11^{2003} ends in 31 $\quad C = 3$ $\quad AB + C^2 = 155(4) + 3^2 = \boxed{629}$

(10)



$A = 8 \cdot 6 \cdot 2 = 96$

$B = 8 \cdot 2 + 6 \cdot 2 + 8 \cdot 2 + 6 \cdot 2 + 8 \cdot 6$

$= 16 + 12 + 16 + 12 + 48 = 104$

$10B - 3A = 10(104) - 3(96) = \boxed{752}$

(11) Pipe 1 $20 \text{ min} \Rightarrow \frac{20}{60}$ full $= \frac{1}{3}$

Pipe 2 $15 \text{ min} \Rightarrow \frac{15}{120} = \frac{1}{8}$

Pipe 3 $10 \text{ min} \Rightarrow \frac{10}{180} = \frac{1}{18}$

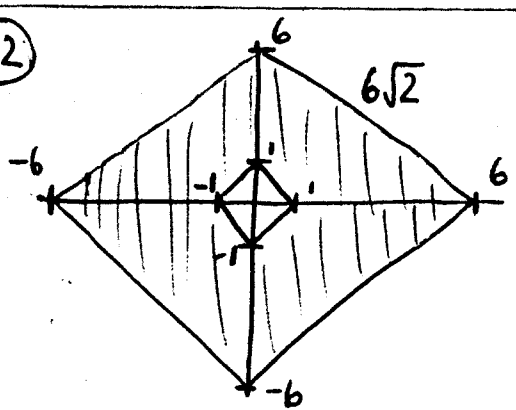
Pipe 4 $5 \text{ min} \Rightarrow \frac{5}{240} = \frac{1}{48}$

tank is now $\frac{1}{3} + \frac{1}{8} + \frac{1}{18} + \frac{1}{48} = \frac{77}{144}$ full

Pipe 5 drains in $\frac{77}{144}(30) = \frac{385}{24} \text{ min} = 962.5 \text{ sec}$

$\boxed{962.5 \text{ or } \frac{1925}{2}}$

(12)



Area of outside square $= (6\sqrt{2})^2 = 72$

Area of inside square $= (\sqrt{2})^2 = 2$

Area of shaded $= 72 - 2 = \boxed{70}$

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(13) $Y = \frac{Kx^2}{Z^3}$

$32 = \frac{K \cdot 16}{27}$

$K = 54$

$Y = \frac{54(6)^2}{(0.5)^3}$

$Y = 15552$

15552

(14) perfect = 120 (30·4)
1 blank = 116 (29·4)
1 wrong = 115 (29·4 - 1)
2 blank = 112 (28·4)
1 blank, 1 wrong = 111 (28·4 - 1)
2 wrong = 110 (28·4 - 2)

6th highest = **110**

(15) B ⇒ 5-25
5-10-10-5
5-10-5-5-5
5-5-5-5-5-5 } 4

C ⇒ 10-10-10
10-10-5-5
10-5-5-5-5 } 3

D ⇒ 25-5 } 1

4 + 3 + 1 = **8**