FAMAT '92

Theta Team Question 1

Line l contains the points $(2, -1)$ and $(7, -8)$.
Line m contains the points $(3, 5)$ and $(9, 4)$.
Lines l and m intersect at point (a, b).
Find the sum $a+b$.

Theta Team Question 2

If $f(x) = \frac{x-1}{x}$, find x such that $f^{-1}(x) = -\frac{1}{2}$.

Theta Team Question 3

Given:

$$X = \begin{bmatrix} 3 & 0 \\ 1 & -2 \\ 5 & -1 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 5 & -2 \\ 1 & 1 \end{bmatrix}$$

A = the a_{21} (second row, first column) entry in Y^{-1}
B = the sum of all entries in the product matrix XY
Find AB.

Theta Team Question 4

Given:

$$\frac{A}{x-4} + \frac{B}{x+3} = \frac{3x+2}{x^2-x-12}$$

$$C+Di = (2+3i)(4-2i) \qquad (\text{where } i = \sqrt{-1})$$

Find $\frac{CD}{AB}$.

Theta Team Question 5

The lengths of the sides of a triangle are 6, 8, and 10. Find the area of the larger of the two triangles into which the bisector of the larger acute angle divides the original triangle.

Theta Team Question 6

Find the sum of all integral solutions of $(x-1)^3(x+2)^3(x-4)^4(x+4)^3 \leq 0$.

Theta Team Question 7

A = the sum of the five arithmetic means between 5 and 47

B = the number of factors of 2 in the 15th term of the geometric sequence 3, 12, 48, . . .

C = the number of terms, n, in the arithmetic sequence whose 1st term is 6, nth term is 129 and common difference is 3

D = The sum of the infinite geometric series whose 1st term is 8 and whose common ratio is $\frac{1}{5}$

Find $\frac{A+B+C}{D}$.

Theta Team Question 8

If b is a positive integer and if $111_b = 223_4$, find b.

Theta Team Question 9

$A = \sqrt{30-\sqrt{30-\sqrt{30-\ldots}}}$

$B = \cfrac{4}{3+\cfrac{4}{3+\cfrac{4}{3+\ldots}}}$

$C = \frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{9}+\frac{1}{8}+\frac{1}{27}+\frac{1}{16}+\frac{1}{81}+\ldots$

$D = 5\cdot\sqrt{5}\cdot\sqrt[4]{5}\cdot\sqrt[8]{5}\ldots$

Find $\frac{ABC}{D}$.

Theta Team Question 10

Consider the hyperbola defined by the equation $4x^2 - 9y^2 - 16x + 18y - 29 = 0$. If (h, k) are the coordinates of the center, c is the distance from the center to a focus, a is the absolute value of the slope of either asymptote and b is the length of the transverse axis, find $\frac{hkc^2}{ab}$.

Theta Team Question 11

A = the number of distinguishable permutations of the letters in the word THETA

B = the number of digits in $2^{10}\cdot 5^{12}$ when written in its usual base 10 form

C = the total number of subsets of $\{1, 3, 5\}$

D = the units digit of 3^{1992}

Find $\frac{AC}{BD}$.

Theta Team Question 12

Given:

$-x + x^2 - x^3 + x^4 - \ldots = \frac{-5}{12}$

Find the value of x.

Theta Team Question 13

A tangent is drawn from $P(6, 1)$ to the circle with equation $x^2 + y^2 + 2x + y - 3 = 0$. Find the distance from P to the point of tangency.

Theta Team Question 14

A tank has a hole in it. Russell alone can completely fill the tank with water in 10 minutes, Richard alone can completely fill it in 4 minutes, and Richard works twice as fast as Russell. Alone, how many minutes would it take the full tank to empty?

Theta Team Question 15

A parabolic tunnel has height 25 meters and width 50 meters. What is the clearance height of a lane whose edges extend 15 meters either side of the center of the tunnel?