

Theta Logs and Exponents Topic Test
FAMAT State Convention 1999

For all questions, answer "E) NOTA" means none of the above.

1. What is the value of $\log_3 35$ to the nearest thousandth?
A) 56.330 B) 0.711 C) 2.209 D) 7.000 E) NOTA
2. What is the units digit of 7998^{1999} ?
A) 2 B) 4 C) 6 D) 8 E) NOTA
3. Solve for x : $\log_8(\log_2(\log_3(\log_2 x))) = 0$
A) 81 B) 100 C) 64 D) 512 E) NOTA
4. $\left(\left(\left(i^{11}\right)^{22}\right)^{33}\right)^{44} =$
A) 1 B) $-i$ C) -1 D) i E) NOTA
5. Simplify, given $a > 0, b > 0$: $\frac{\sqrt{a} + \sqrt{b}}{a^{\frac{-1}{2}} + b^{\frac{-1}{2}}}$
A) $a + b + \sqrt{ab}$ B) 1 C) \sqrt{ab} D) $\frac{\sqrt{ab}}{a+b}$ E) NOTA
6. Solve for x : $8^{2x^2} = 32^{\frac{4}{3}x+7}$
A) $\{\frac{7}{2}\}$ B) $\{1,0\}$ C) $\{-\frac{7}{3}, \frac{5}{2}\}$ D) $\{\frac{5}{2}\}$ E) NOTA

7. Which of the following expressions are equivalent to each other?

I. $2\log\left(\frac{2x}{\sqrt{3}}\right)$ II. $\log\left(\frac{2x^2}{\sqrt{3}}\right)$ III. $\log\left(\frac{4}{3}\right) + \log(x^2)$ IV. $\frac{\ln\left(\frac{4x^2}{\sqrt{3}}\right)}{\ln(10)}$ V. $\frac{4}{3}\log(x)$

- A) I, II B) I, III C) III, IV, V D) II, IV E) NOTA

8. Express $\log_3 16200$ in terms of p , q , r , and s , if $p = \log 2$, $q = \log 5$, $r = \log 7$, and $s = \log 3$.

A) $\frac{3p+2q+4s}{s}$ B) $\frac{2q+3r+4}{s}$ C) $4+3p+2q$ D) $\frac{3q}{s}+4p$ E) NOTA

9. At the end of their sophomore year, Mark, James, Jessica, and Cathy deposit \$96.48, \$90.00, \$100.00, and \$102.00 respectively into new bank accounts. If they don't withdraw or deposit anymore money until they graduate from high school (in exactly two years), and their continuously compounded interest rates are (in the same order as before) 8.5%, 9.5%, 6%, and 5%, who will have made the most profit by graduation day?

- A) Mark B) James C) Jessica D) Cathy E) NOTA

10. Solve for x : $\log_4(-x^2 - 4x + 21) = \log_2(x + 1)$

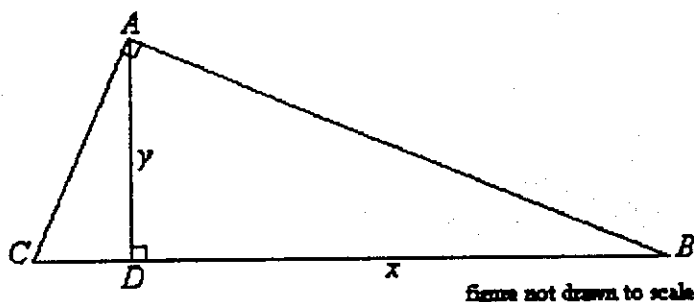
- A) 2 B) -5 C) 10 D) 4 E) NOTA

11. Solve for a :

$$\begin{vmatrix} \log(8a^3) & \log(4a) \\ \log_2 16 & \log_3 81 \end{vmatrix} = 0$$

- A) $4\sqrt{2}$ B) $2\sqrt{2}$ C) $\frac{\sqrt{2}}{2}$ D) $\frac{\sqrt{2}}{4}$ E) NOTA

12. If $\log_2 x = 3$ and $\log_y 36 = 2$, then find the area of right triangle ABC whose altitude AD is of length y and segment DB is of length x , as shown below.



- A) 8 B) 36 C) 42 D) 37.5 E) NOTA
13. Which function grows the fastest as x gets infinitely large?
- $f(x) = 2^x$ $g(x) = \log_2 x$ $h(x) = x^2$
- A) $f(x)$ B) $g(x)$ C) $h(x)$ D) $g(f(x))$ E) NOTA
14. Simplify:
- $$\log_{\log_{\log_{\dots 27} 27} 27}$$
- A) $\sqrt{3}$ B) $3\sqrt{3}$ C) 3 D) 9 E) NOTA
15. The number of hours Gill puts off doing his homework is inversely proportional to some power of the number of assignments he has. On the rare occasion that Gill only has one assignment, he procrastinates for nine hours. But when he has eight assignments, he only stalls for two hours and fifteen minutes. On a typical night, when Gill has 125 homework assignments, how many minutes does he waste before starting his work?

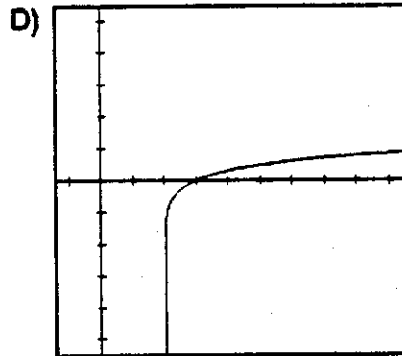
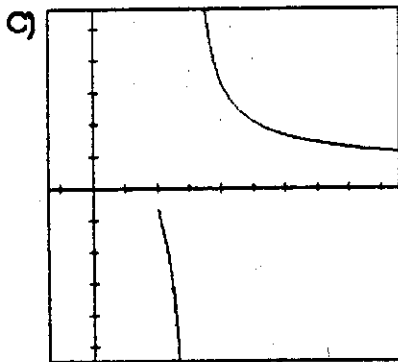
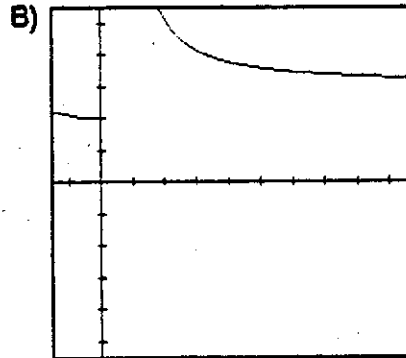
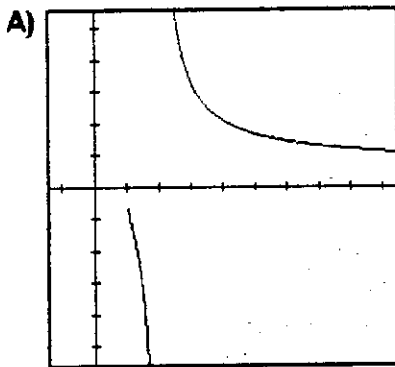
- A) .36 B) 21.6 C) 33.75 D) 67.2 E) NOTA

16. Put the following in order from least to greatest:

I. 1 II. 2^{-1} III. $\log 12$ IV. $\left(\frac{e}{5}\right)^x$

A) I, II, IV, III B) II, I, IV, III C) IV, II, I, III D) II, I, IV, III E) NOTA

17. Which of the following most closely represents the inverse of the graph of $y = 10^{\frac{1}{x}} + 2$? (note: all tick marks represent one unit)



E) NOTA

18. If $x = t \log_2 4t$ and $y = 2^{\log_2 t}$, then find the value of x when $y = 4$.

A) 64 B) 96 C) 98 D) 256 E) NOTA

19. How many digits are in 3^{99} ?
- A) 47 B) 48 C) 49 D) 50 E) NOTA
20. $\prod_{n=2}^{31} \log_{(n+1)} n =$
- A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) NOTA
21. What is the sum of the coefficients of the expansion of $(2x + 3y - 4z + 1)^6$?
- A) 2 B) 32 C) 0 D) 64 E) NOTA
22. Simplify: $(2\sqrt{2}x^3y^{\frac{1}{2}}z^4)^{-2}(4\sqrt{3}x^5y^3z^{\frac{2}{3}})$
- A) $32\sqrt{3}x^{11}y^4z^{\frac{26}{3}}$ B) $\frac{2\sqrt{3}xz^{\frac{22}{3}}}{3y^2}$ C) $\frac{\sqrt{3}y^2}{2xz^7}$ D) $\frac{\sqrt{6}z^{\frac{10}{3}}}{6x^2y^{2.5}}$ E) NOTA
23. If $c^d = d^c$, what is the value of $\log_c d$?
- A) $\frac{d}{c}$ B) $\frac{c}{d}$ C) cd D) d E) NOTA
24. What is the sum of the solutions of $\log_2(x^2 + 1) - \log_2(2x - 1) = 2$?
- A) 0 B) $4 + \sqrt{11}$ C) $2\sqrt{11}$ D) 8 E) NOTA
25. $\exp\left(\sum_{x=1}^n \ln(x)\right) =$
- A) $\ln(n)$ B) e^n C) $e^{n!}$ D) $\ln(n!)$ E) NOTA

26. What is the characteristic of $\log_{18} 1980$?

- A) 1 B) 2 C) 3 D) 4 E) NOTA

27. What is the domain of the function $f(x) = \log(x^2 - 4) - \log(x - 2)$?

- A) $[-2, \infty)$ B) $[-2, 2) \cup (2, \infty)$ C) $(2, \infty)$ D) $(-\infty, \infty)$ E) NOTA

28. Simplify: $\ln\left(\frac{6x^2y^{\frac{1}{2}}z^{\frac{1}{3}}}{16^{\frac{1}{4}}x^3z^{-\frac{2}{3}}}\right) - \ln\left(\frac{3x^{-1}z}{5y^{\frac{1}{2}}}\right)$

- A) $\ln\left(\frac{9yz^2}{5x^2}\right)$ B) $\ln(5)$ C) $\ln(5\sqrt[3]{z^4})$ D) $2\ln\left(\frac{3z\sqrt{y}}{x}\right)$ E) NOTA

29. What is the third term in the expansion of $(x + 2)^{-4}$?

- A) $80x^{-6}$ B) $24x^2$ C) $20x^{-6}$ D) $160x^2$ E) NOTA

30. Solve the system of equations:

$$\log_2 x = \frac{y}{2}$$

$$\log_4\left(\frac{7}{2}x + 4\right)^2 = y - 1$$

- A) (2,4) B) (4,2) C) (6,8) D) (8,6) E) NOTA