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FAMAT State Convention 1999 Solutions
Logs & Exponents, Theta Level

① $\log_5 35 = \frac{\log 35}{\log 5} \approx 2.207$ $\boxed{C}$

② The units digit follows the pattern: 8, 4, 2, 6, 8, 4, 2, 6...

1999 ÷ 4 = 499.75, so the units digit will be the third in the pattern, which is 2. $\boxed{A}$

③ $\log_8(\log_2(\log_3(\log_2 x))) = 0$

$\log_2(\log_3(\log_2 x)) = 8^0 = 1$

$\log_3(\log_2 x) = 2^1 = 2$

$\log_2 x = 3^2 = 9$

$x = 2^9 = 512$ $\boxed{D}$

④ $(((i^{11})^{22})^{33})^{44} = i^{351384} = 1$ $\boxed{A}$

351384 mod 4 = 0

⑤ $\frac{\sqrt{a}+\sqrt{b}}{a^{-1/2}+b^{-1/2}} = \frac{\sqrt{a}+\sqrt{b}}{\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}} = \frac{\sqrt{a}+\sqrt{b}}{\frac{\sqrt{a}+\sqrt{b}}{\sqrt{ab}}}$

$\frac{\sqrt{a}+\sqrt{b}}{\frac{\sqrt{a}+\sqrt{b}}{\sqrt{ab}}} \cdot \frac{\frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}}{\frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}}} = \sqrt{ab}$ $\boxed{C}$

⑥ $8^{2x^2} = 32^{\frac{11}{5}x+7}$

$(2^3)^{2x^2} = (2^5)^{\frac{11}{5}x+7}$

$6x^2 = 11x + 35$

$6x^2 - 11x - 35 = 0$

$(2x-7)(3x+5) = 0$

$x = \left\{\frac{7}{2}, \frac{-5}{3}\right\}$ $\boxed{E}$

⑦ $2\log\left(\frac{2|x|}{\sqrt{3}}\right) = \log\left(\frac{2x}{\sqrt{3}}\right)^2 = \log\left(\frac{4}{3}x^2\right) =$

$\log\left(\frac{4}{3}\right) + \log(x^2)$

I, III $\boxed{B}$

⑧ $\log_3 16200 = \frac{\log 16200}{\log 3} = \frac{\log(2^3 \cdot 3^4 \cdot 5^2)}{\log 3} =$

$\frac{3\log 2 + 4\log 3 + 2\log 5}{\log 3} = \frac{3p+2q+4s}{s}$ $\boxed{A}$

⑨ $A = Pe^{rt}$; $t = 2$ for all cases

	P	r	A	Profit		
Mark	\$96.48	8.5%	\$114.36	\$17.88		
James	\$90.00	9.5%	\$108.83	\$18.83	*	$\boxed{B}$
Jessica	\$100.00	6%	\$112.75	\$12.75		
Cathy	\$102.00	5%	\$112.73	\$10.73		

⑩ $\log_4(-x^2-4x+21) = \log_2(x+1)$

$\frac{\log(-x^2-4x+21)}{\log 4} = \frac{\log(x+1)}{\log 2}$

$\log(-x^2-4x+21) = \frac{\log(x+1)\cdot\log 4}{\log 2}$

$\log(-x^2-4x+21) = \log(x+1)\cdot\log_2 4$

$\log(-x^2-4x+21) = 2\log(x+1) = \log(x+1)^2$

$-x^2-4x+21 = x^2+2x+1$

$0 = 2x^2+6x-20$

$0 = 2(x+5)(x-2)$

$x = \cancel{-5}, 2$ $\boxed{A}$

⑪ $\begin{vmatrix} \log 8a^3 & \log 4a \\ \log_2 16 & \log_3 81 \end{vmatrix} = 0$ $\log_2 16 = \log_3 81 = 4$

$4(\log 8a^3 - \log 4a) = 0$

$\log\left(\frac{8a^3}{4a}\right) = \log 2a^2 = 0$

$2a^2 = 1$ $a^2 = \frac{1}{2}$

$a = \frac{\sqrt{2}}{2}$ $\boxed{C}$

12. $\log_2 x = 3 \quad x = 2^3 = 8$

$\log_y 36 = 2 \quad y^2 = 36 \quad y = 6$

$\text{Area} = \frac{1}{2}(AD)(BC)$

$\frac{BD}{AD} = \frac{AD}{CD} \quad 8(CD) = 6^2 \quad CD = \frac{9}{2}$

$\text{Area} = \frac{1}{2}(6)(8 + \frac{9}{2}) = 37.5 \quad \boxed{D}$

13. $g(f(x)) = \log_2 2^x = x$

$f(x) = 2^x$ grows fastest $\boxed{A}$

14. $\log_x 27 = x$

$x^x = 27 \quad x = 3 \quad \boxed{C}$

15. $y = k\left(\frac{1}{x^n}\right) \quad k = 9$ from $(1,9)$

$\frac{9}{4} = 9\left(\frac{1}{8^n}\right) \quad 8^n = 4 \quad n = 2/3$

$y = 9x^{-2/3} \qquad y = 9(125)^{-2/3} = \frac{9}{25}$

$\frac{9}{25} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 21.6$ minutes

$\boxed{B}$

16. I. 1

II. $2^{-1/2} \approx .707$

III. $\log 12 \approx 1.079$

IV. $\left(\frac{e}{5}\right)^{\pi} \approx .1474$

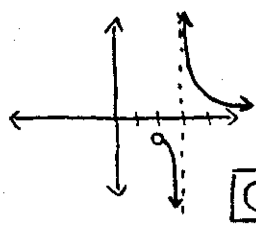
IV, II, I, III $\boxed{C}$

17. $x = 10^{\frac{1}{y}} + 2$

$10^{\frac{1}{y}} = x - 2$

$\log(x-2) = \frac{1}{y}$

$y = \frac{1}{\log(x-2)}$



$\boxed{C}$

18. $4 = 2^{\log_4 t} \quad 2 = \log_4 t \quad 4^2 = t \quad t = 16$

$x = 16 \log_2(4 \cdot 16) = 96 \quad \boxed{B}$

19. $\log 3^{99} = 99 \log 3 \approx 47.24 \Rightarrow$

$\boxed{B}$ 48 digits

20. $\prod_{n=2}^{31} \log_{(n+1)} n = \log_3 2 \cdot \log_4 3 \cdot \log_5 4 \ldots \log_{32} 31$

$\frac{\log 2}{\cancel{\log 3}} \cdot \frac{\cancel{\log 3}}{\cancel{\log 4}} \cdot \frac{\cancel{\log 4}}{\cancel{\log 5}} \cdots \frac{\cancel{\log 30}}{\cancel{\log 31}} \cdot \frac{\cancel{\log 31}}{\log 32}$

$\frac{\log 2}{\log 32} = \frac{1}{5} \quad \boxed{D}$

21. $(2+3-4+1)^6 = 2^6 = 64 \quad \boxed{D}$

22. $(2\sqrt{2}\, x^3 y^{1/2} z^4)^{-2}(4\sqrt{3}\, x^5 y^3 z^{2/3}) =$

$\frac{4\sqrt{3}\, x^5 y^3 z^{2/3}}{8x^6 y z^8} = \frac{\sqrt{3}\, y^2}{x z^{22/3}} \quad \boxed{E}$

23. $\log_c d = x \quad c^x = d$

$c^d = d^c \quad c^{d/c} = d^{c/c} = d \quad x = \frac{d}{c} \quad \boxed{A}$

24. $\log_2\left(\frac{x^2+1}{2x-1}\right) = 2$

$\frac{x^2+1}{2x-1} = 4 \quad x^2 + 1 = 8x - 4$

$x^2 - 8x + 5 = 0$

$x = \frac{8 \pm \sqrt{44}}{2} = 4 \pm \sqrt{11} \quad \text{sum} = 8 \quad \boxed{D}$

25. $\exp\left(\sum_{x=1}^{n} \ln x\right) = e^{\ln 1 + \ln 2 + \ln 3 \ldots + \ln(n)} =$

$e^{\ln(n!)} = n! \quad \boxed{E}$

26. $\frac{\log 1980}{\log 18} \approx 2.626 \Rightarrow 2 \quad \boxed{B}$

27. $x^2 - 4 > 0$ and $x - 2 > 0$

$x^2 > 4 \qquad x > 2$

$|x| > 2 \qquad (2, \infty) \quad \boxed{C}$

(28) $\ln\left(\frac{6x^2y^{1/2}z^{1/3}}{2x^3z^{-2/3}}\cdot\frac{5y^{-1/2}}{3x^{-1}z}\right) = \ln 5$ $\boxed{B}$

(29) $(x+2)^{-4} =$

$x^{-4} + -4(x^{-5})(2^1) + 20x^{-6}(2^2) \ldots =$

$x^{-4} - 8x^{-5} + 80x^{-6}$ E

(30) $\begin{cases} \log_2 x = \frac{y}{2} \\ \log_4(\frac{7}{2}x+4)^2 = y-1 \end{cases}$

$\log_2 x^2 = y$

$\log_4(\frac{7}{2}x+4)^2 + \log_4 4 = y$

$\log_2 x^2 = \log_4(7x+8)^2$

$\frac{\log x^2}{\log 2} = \frac{2\log(7x+8)}{2\log 2}$

$x^2 = 7x+8 \qquad x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$x = 8, \cancel{-1}$

$\log_2 8^2 = y$

$\log_2 64 = y \quad y = 6$

$(8,6)$ $\boxed{D}$