LOGS + EXPONENTS SOLUTIONS

(1) $\log_4\left(\frac{1}{7}\right) = \log_4 1 - \log_4 7$
$= 0 - N = -N$ (C)

(2) FROM CALCULATOR:
$10^{1.4349} = 27.2$ (D)

(3) $3^2 = x^2 - 8x$
$x^2 - 8x - 9 = 0$ (A)
$(x-9)(x+1) = 0$
$x = 9, -1$

(4) $2^x = 3^{x+1}$
$x\log 2 = (x+1)\log 3$
$x\log 2 = x\log 3 + \log 3$ (C)
$x\log 2 - x\log 3 = \log 3$
$x(\log 2 - \log 3) = \log 3$
$x = \frac{\log 3}{\log 2 - \log 3}$

(5) $\log_b a = 2c + \log_b c$
$\log_b a - \log_b c = 2c$
$\log_b \frac{a}{c} = 2c$
$b^{2c} = \frac{a}{c}$ (D)
$a = cb^{2c}$

(6) $2^{3x} = 4^{x+1}$
$2^{3x} = (2^2)^{x+1} = 2^{2x+2}$ (E)
$3x = 2x + 2$
$x = 2$

(7) $(81)^{3/4} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$ (B)

(8) $\log_a 12 = \log_a 2^2 \cdot 3$
$= 2\log_a 2 + \log_a 3$
(C) $= 2(.477) + .301 = 1.255$

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(9) $\frac{\log m}{\log n} \neq \log m - \log n$
$\log m^{\frac{1}{2}} = \frac{1}{2}\log m = \log m^{\frac{1}{2}}$
$\log 2x \neq 2\log x \neq \log x^2$
(B)

(10) $AE = A = \frac{d_1 d_2}{2} = \frac{N(N+3)}{2} = \frac{N^2+3N}{2}$
(B)

(11) $\frac{4^{-1}x^{4/9}y^{-14/12}}{2x^{-8/9}y^{6/12}} = \frac{x^{6/9}}{2\cdot 4\cdot y^{20/12}}$
$= \frac{x^{2/3}}{8y^{5/3}}$ (C)

(12) $x\left(\frac{\frac{1}{x}-\frac{1}{y}}{x^2-y^2}\right) = x\left(\frac{y-x}{xy(x^2-y^2)}\right)$
$= \frac{y-x-1}{y(x-y)(x+y)} = \frac{-1}{y(x+y)}$ (D)

(13) $\log e^{\frac{i}{I}} = -t/RC$
$e^{-t/RC} = i/I$
$i = Ie^{-t/RC}$ (C)

(14) $2\log x - \log(1-2x) = 1$
$\log \frac{x^2}{1-2x} = 1$
$10 = \frac{x^2}{1-2x}$
$10 - 20x = x^2$
$x^2 + 20x - 10 = 0$
$x = \frac{-20 \pm \sqrt{440}}{2}$
$x = \frac{-20 \pm 2\sqrt{110}}{2}$
$= -10 \pm \sqrt{110}$
$x = \sqrt{110} - 10$ (E)

(15) $(2x-5)^4 = (2x)^4 - 4(2x)^3(5) + 6(2x)^2(5)^2 + \cdots$
$6(2x)^2(5)^2 = 24(25x^2) = 600x^2$
600 (A)

(16) $\log_9\left(\frac{1}{27}\right) = \frac{\log\left(\frac{1}{27}\right)}{\log 9} = -1.5 = -\frac{3}{2}$ (C)

(17) $(a+\sqrt{a^2-1})^3 = a^3 + 3a^2\sqrt{a^2-1} + 3a(a^2-1) + (a^2-1)^{3/2}$

$(a-\sqrt{a^2-1})^3 = a^3 - 3a^2\sqrt{a^2-1} + 3a(a^2-1) - (a^2-1)^{3/2}$ E

ADD, WE GET $2a^3 + 6a(a^2-1)$ (A)

(18) $(1+x)^{-2} = 1^{-2} - 2(1^{-3})x - \frac{2(-3)}{2}(1^{-4})x^2 - \frac{2(-3)(-4)}{2\cdot 3}(1^{-5})x^3 + \cdots$

4th term $\frac{-2(-3)(-4)}{2\cdot 3}(1^{-5})x^3 = -4x^3$ (B)

(19) $x\log 5^2 = (x+1)\log 7$

$x\log 5^2 = x\log 7 + \log 7$

$x\log 25 - x\log 7 = \log 7$

$x = \frac{\log 7}{\log 25 - \log 7}$ (B)

(20) $\log\frac{40x-1}{x-1} = 3$

$1000 = \frac{40x-1}{x-1}$

$1000x - 1000 = 40x - 1$

$960x = 999$

$x = 999/960$

$x = 333/320$ (A)

(21) $x\log bc^2 = x\log b^3 + \log b$

$x\log bc^2 - x\log b^3 = \log b$

$x = \frac{\log b}{\log bc^2 - \log b^3}$ (E)

$= \log b / (\log bc^2 - 3\log b)$

(22) $\log(x^2+3x) + \log 5x - \log 2x = 1$

$\log\frac{(x^2+3x)5x}{2x} = 1$

$10 = \frac{5}{2}(x^2+3x)$

$20 = 5(x^2+3x)$

$4 = x^2+3x$

$0 = x^2+3x-4$

$0 = (x+4)(x-1)$

$x = 1, -4$

B

(23) $(3^3)^{x^2+1} = 3^5$

$3^{3x^2+3} = 3^5$

$3x^2+3 = 5$

$3x^2 = 2$

$x^2 = 2/3$

$x = \pm\sqrt{2/3}$ (A)

(24) $e^{-Rt/L} = 1 - \frac{RI}{E}$

$-Rt/L(\ln e) = \ln\left(1 - \frac{RI}{E}\right)$

$t = -\frac{L}{R}\left(\ln\left(1 - \frac{RI}{E}\right)\right)$ (A)

(25) $\log_{.3} x < \log_{.3} 4/3$

$x\log.3 < \log.4 - \log.3$

$x < \frac{\log.4 - \log.3}{\log.3}$

USING CALCULATOR + SINCE $\log.3 < 0$, $x > -.239$ (C)

(26) $(x+y)\log 5 = 2$ $(2x-y)\log 2 = 1$

$x+y = \frac{2}{\log 5}$

$2x-y = \frac{1}{\log 2}$

ADD $3x = \frac{2}{\log 5} + \frac{1}{\log 2}$

$x = \left(\frac{2}{\log 5} + \frac{1}{\log 2}\right)\frac{1}{3} \doteq 2.06$

$\therefore y \doteq .80$ $(2.06, .80)$ (A)

(27) N = # ORGANISMS

$N = 2^{30} = 1.073741824$

$= 1.07 \times 10^9$ (A)

(28) x = SMALLEST, $x+1$ = MIDDLE

$x+2$ = LARGEST

$(x+1)^2 - 5(x+2) = 3x+7$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$, $x+1 = 9$, $x+2 = 10$ (D)

(29) $\frac{7x^{-1}}{x^{-3}+y^{-4}} = \frac{7/x}{1/x^3 + 1/y^4} = \frac{7}{1/x^2 + x/y^4} = \frac{7x^2y^4}{y^4+x^3}$ (B)

(30) $8 + 5 - \frac{1}{24} + \left(\frac{x^{1/8}\cdot 8}{2}\right)(3x^{-9/8})$

$13 - \frac{1}{24} + 12x^{-8/8}$

$12x^{3/4} + 12/x = 5\frac{1}{4} + 12/x$ (A)