17 LOGS + EXPONENTS STATE CONVENTION 1993 SOLUTIONS

1)
I $y = \log_3 12$
II $y = \log_3 12$
III $y = \log_3 4 + 1$
$= \log_3 4 + \log_3 3$
$= \log_3 12$
$\therefore$ D

2)
$\frac{\log X}{\log 2} = \frac{\log 81}{\log 4}$
$\log X = \frac{\log 81}{\log 4} \cdot \log 2$
$\log X = \log 81 (\log_4 2)$
$\log X = \frac{1}{2}(\log 81)$
$\log X = \log 9$
$X = 9 \quad \therefore$ C

3)
$\log_8(-8) = x$
$8^x = -8$
$\varnothing \quad \therefore$ E

4)
$\log_2 8 = 3$
$\therefore$ C

5) The graphs of $y = \log x$ and $y = \ln x$ intersect at only $(1, 0)$.
$\therefore$ C

6) $\log_5\left(\frac{8}{1000}\right) = \log_5\left(\frac{1}{...}\right)$
$= -3 \quad \therefore$ b

7)
$\frac{1}{\frac{\log N}{\log 2}} + \frac{1}{\frac{\log N}{\log 3}} + \cdots + \frac{1}{\frac{\log N}{\log 25}}$
$\frac{\log 2}{\log N} + \frac{\log 3}{\log N} + \cdots + \frac{\log 25}{\log N}$
$\frac{\log(2 \cdot 3 \cdots 25)}{\log N} = \log_N 25!$
$\therefore$ d

8) $\frac{5}{3} \quad \therefore$ d

9)
I $\log(4 \cdot 16) = \log 64$
II $\log\left(\frac{128}{2}\right) = \log 64$
III $\log(2^6) = \log 64$
IV $\log(4^3) = \log 64$
V $\log 8^2 = \log 64 \quad \therefore$ d
VI $\log 64 \quad \therefore$ Prob $= 1$

10)
$\log(x^{\log x}) = \log(100x)$
$\log^2 x - \log x - \log 100 = ...$
$(\log x)^2 - \log x - 2 = 0$
$(\log x - 2)(\log x + 1) = 0$
$\log x = 2 \quad \log x = -1$
$x = 100 \quad x = 1/10$
$100 \cdot 1/10 = 10 \quad \therefore$ C

11)
$\log\left(\frac{108}{5}\right) = \log\left(\frac{216}{10}\right)$
$\log(2^3 \cdot 3^3 \div 10)$
$3\log 2 + 3\log 3 - \log 10$
$3c + 3d - 1 \quad \therefore$ d

12)
$\log_{x^2+6}(10) = 1$
$x^2 + 6 = 10$
$x^2 = 4$
$x = \pm 2$
$\therefore$ d

13)
$6^{x^2} = 6^{-3}$
$x^2 - 2 = -3$
$x^2 = -1$
$\therefore$ E

14) $2^1 a^{1/3} + 4 - 3 + 3 - 1 = 2a^{1/3} + ...$
$\therefore$ d

15) $(3758x)^0 = 1$
$\therefore$ b

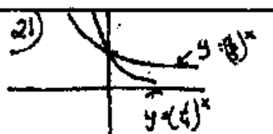
16)
$(\log x)^2 = \log x$
$(\log x)^2 - \log x = 0$
$\log x(\log x - 1) = 0$
$x = 1 \quad x = 10$
$1 + 10 = 11 \quad \therefore$ d

17)
$2^{2x} - 4 \cdot 2^x - 32 = 0$
$(2^x - 8)(2^x + 4) = 0$
$2^x = 8 \quad 2^x = -4$
$x = 3$
$\therefore$ C

18)
$xe^{kt} = ye^{mt}$
$\frac{x}{y} = e^{mt - kt}$
$mt - kt = \ln\left(\frac{x}{y}\right)$
$t = \frac{\ln x - \ln y}{m - k} \quad \therefore$ a

19)
$2\log\left[\frac{1}{3}(a+b)\right] = \log(ab)$
$\log\left[\frac{1}{3}(a+b)\right]^2 = \log(ab)$
$\log \frac{1}{9}(a+b)^2 = \log(ab)$
$(a+b)^2 = 9ab$
$a^2 + 2ab + b^2 = 9ab$
$a^2 + b^2 = 7ab \quad \therefore$ a

20)
$6^x + 6^{-x} - \frac{10}{3} = 0$
$6^{2x} + 1 - \frac{10}{3}6^x = 0$
$3 \cdot 6^{2x} - 10 \cdot 6^x + 3 = 0$
$(3 \cdot 6^x - 1)(6^x - 3) = 0$
$6^x = 1/3 \quad 6^x = 3$
$x = \log_6(3^{-1}) \quad x = \log_6 3$
$x = \frac{-\log 3}{\log 6} \quad x = \frac{\log 3}{\log 6}$
$x = \frac{-b}{a+b} \quad x = \frac{b}{a+b}$
$\therefore$ b

21)


$(-\infty, 0) \quad \therefore$ b

22) $81^{-1/4} = 1/3$
$\therefore$ C

23)
$2^{x-4} = 2^3 \Rightarrow x = 7$
$3^{2x-5y} = 3^5 \Rightarrow$
$y = 17/5$
$xy = \frac{119}{5} \quad \therefore$ d

24)
$e^2(e^{2x} + e^x - 6) = 0$
$(e^x + 3)(e^x - 2) = 0$
$e^x = -3 \quad e^x = 2$
$x = \ln 2$
$\therefore$ b

25)
$e^{\ln\left(\frac{x^2}{x+2}\right)} = \frac{8}{3}$
$\frac{x^2}{x+2} = \frac{8}{3}$
$3x^2 - 8x - 16 = 0$
$(3x + 4)(x - 4) = 0$
$x = -4/3 \quad x = 4 \quad \therefore$ b

26) $3^{\log_3 16} = 16$
$\therefore$ d

27)
$\frac{4!}{2!2!}(2x)^2(-y)^2$
$6(4x^2)(y^2)$
$24x^2y^2$
$\therefore$ C

28)
$\log_{12} 12^{1/12}$
$\frac{1}{12}\log_{12} 12$
$\frac{1}{12}(1)$
$\frac{1}{12}$
$\therefore$ b

29)
$2^k(2^2 - 1) = 192$
$2^k(3) = 192$
$2^k = 64$
$k = 6$

30)
$f(2^{2\log_2(2)}) = f(2^2) = f(4)$
$f(4) = \log_2(4) = 2$
$\therefore$ C