2001 State Mu Topic Test Solutions – Circles, Perimeter, Area, and Volume

1. $\pi d = C$, $\pi d = \pi$, therefore d = 1. Using 45-45-90 rule find the side length to be $s = \frac{\sqrt{2}}{2}$.

   Perimeter = 4(s) = $4\left(\frac{\sqrt{2}}{2}\right) = \boxed{2\sqrt{2} = C}$.

2. V = Bh = $6\left(\frac{s^2\sqrt{3}}{4}\right)(h) = 6\left(\frac{5^2\sqrt{3}}{4}\right)(12) = \boxed{450\sqrt{3} = D}$

3. The two diagonals bisect forming a triangle with legs of 3 and 4. The hypotenuse (also the side of the rhombus) is therefore 5. Perimeter = 4*s* = 4(5) = $\boxed{20 = B}$.

4. Use three ratios: $\frac{a}{x} = \frac{x}{a+b}, x = AD; -\frac{a}{y} = \frac{y}{b}, y = BD; -\frac{b}{z} = \frac{z}{a+b}, z = DC$. Solve for {*x*,*y*,*z*}.

   $\left\{x = \sqrt{a^2 + ab}, y = \sqrt{ab}, z = \sqrt{ab + b^2}\right\}$. Perimeter = $\boxed{a + b + 2\sqrt{ab} + \sqrt{a^2 + ab} + \sqrt{ab + b^2} = A}$

5. $V_{cube} - V_{cylinder} = s^3 - \pi r^2 h = 8^3 - (8\pi(4^2)) = \boxed{512 - 128\pi = D}$

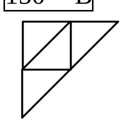
6. C = $2\pi r$; P = $6r\sqrt{3}$ (Using 30-60-90 rule, side length = $2r\sqrt{3}$). $\frac{C}{P} = \frac{2\pi r}{6r\sqrt{3}} = \boxed{\frac{\pi\sqrt{3}}{9} = A}$

7. $A_c = \pi r^2$; $A_t = \frac{s^2\sqrt{3}}{4}$. $\frac{A_c}{A_t} = \frac{\pi r^2}{\left(\frac{(2r\sqrt{3})^2\sqrt{3}}{4}\right)} = \boxed{\frac{\pi\sqrt{3}}{9} = A}$

8. $A_{semi\text{-}cir.} - A_{small\text{-}cir.} = \left(\frac{1}{2}\pi 16^2\right) - (\pi 8^2) = \boxed{64\pi = C}$

9. $A_{hexagon} - A_{triangle} = \left(\frac{(6)6^2\sqrt{3}}{4}\right) - \left(\frac{(6\sqrt{3})^2\sqrt{3}}{4}\right) = \boxed{27\sqrt{3} = A}$

10. $B_1 = 10\sqrt{2}$, $B_2 = 10\sqrt{2} + 10\sqrt{2}$ using 45-45-90 rule. $A_{trap.} = \frac{1}{2}(B_1 + B_2)h = \frac{1}{2}(10\sqrt{2} + 20\sqrt{2})\left(\frac{10}{\sqrt{2}}\right) = \boxed{150 = B}$



11. $A_{tri.} = 4\sqrt{3} = \frac{s^2\sqrt{3}}{4}; s = 4$. $A_{s1} = (2\sqrt{3})^2 = 12$. $A_{s2} = 4^2 = 16$. 12:16 = $\boxed{3:4 = C}$

12. Herons Formula; $A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{10(10-5)(10-7)(10-8)} = \sqrt{300} = \boxed{10\sqrt{3} = B}$

13. $V_{cylinder} = \pi r^2 h = \pi 2^2(6) = \boxed{24\pi = B}$

14. $V_{bicone} = 2\left(\frac{1}{3}\pi r^2 h\right)$; use 45-45-90 to find *r* and *h*. $2\left(\frac{1}{3}\pi\left(\frac{3}{\sqrt{2}}\right)^2\left(\frac{3}{\sqrt{2}}\right)\right) = \boxed{\frac{9\pi\sqrt{2}}{2} = D}$

15. $V_{box} = (11 - 2(2))(8.5 - 2(2))(2) = \boxed{63 = A}$

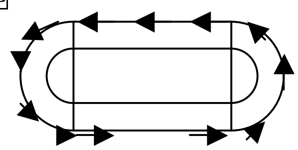
16. $A_{sector} = \left(\frac{m}{360}\right)p\,r^2 = \left(\frac{11.25+11.25}{360}\right)p\,4^2 = \boxed{p = D}$

17. In order to elimate the "open space" created when filling the vase the marbles should be smaller and smaller to minimize the dead space, therefore the radius must become increasingly small, or approach zero, $\boxed{A}$.

18. The polygon formed results in the minimum distance; 6 (distance) x 8 (number of gaps) = $\boxed{48 = B}$.

19. Circum. = $4p = pd$; r=2. $A_{segment} = A_{sector} - A_{triangle} = \left(\frac{1}{4}p\,2^2\right) - \left(\frac{1}{2}(2)(2)\right) = \boxed{p - 2 = A}$

20. Show factors of sides as in the table below. Then match up the sides; F=14x8, T=14x6, S=6x8. Therefore volume is (14)(8)(6) = $\boxed{672 = C}$



Front	Top	Side
(1)(112)	(1)(84)	(1)(48)
…	…	…
(8)(14)	(6)(14)	(6)(8)
…	…	…

21. The track is pictured above. Runner B follows the path shown with arrow, while A runs the inside track. The straight distances are the same, but the radii of the circle run by B is larger. $R_{inside\ track} = \frac{160}{p}$

$$d_a = 1000 + 160 + 160. - d_b = 1000 + 2p\left(\frac{160}{p} + 15\right). \quad d_b - d_a = \boxed{30p = C}$$

22. $A_{rhombus} = \frac{1}{2}d_1 d_2 = \frac{1}{2}(10)(24) = \boxed{120 = C}$.

23. $A_{cir.}$=64π;r=8(also triangle altitude). Use 30-60-90 rules to find $s_{tri.} = \frac{16\sqrt{3}}{3}$. $A_{tri} = \frac{\left(\frac{16\sqrt{3}}{3}\right)^2\sqrt{3}}{4} = \boxed{\frac{64\sqrt{3}}{3} = C}$

24. Smallest side of the large triangle=15. Use triangle similarity ratios to find side lengths of 15, 26.25, and 18.75. Σ = $\boxed{60 = D}$.

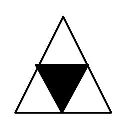
25. r = 4; circum.= 8π. $8p = 4s$. Side length of square = 2π. $L_{diag.} = \sqrt{(2p)^2 + (2p)^2} = \sqrt{8p^2} = \boxed{2p\sqrt{2} = D}$.

26. Area of the hexagon: $(6)\frac{s^2\sqrt{3}}{4}$; s=x. The area of one section would be $(6)\frac{s^2\sqrt{3}}{4}\left(\frac{1}{3}\right) = \frac{x^2\sqrt{3}}{2} = C$

27. A_{circle}=36π; r=6. Use 30-60-90 to find radius of large circle (12). $A_{large\ cir.} = 12^2\pi = \boxed{144\pi = D}$

28. Eulers Formula: V – E + F = 2, 18 – 26 + F = 2, F = 10. If each faces has S.A. of 3, then total S.A. = (10)(3) = $\boxed{30 = C}$.

29. The new triangle splits the original into 4. The ratio is therefore $\boxed{1:4 = B}$.



30. $L_{arc} = \left(\frac{m}{360}\right)(2pr) = \left(\frac{180 + 30\left(20/60\right)}{360}\right)(2p(2)) = \boxed{\frac{19p}{9} = B}$