Solutions Dig

D (1) I. $3{,}960° \equiv 0°$
II. $-9{,}000° \equiv 0°$
III. $30° = 30°$
IV. $1530° \equiv 90°$ $\quad \therefore$ I & II only

E (2) $\sec 60° \cdot \tan 300° \cdot \cos 300° \cdot \sin 30° =$
$2 \cdot -\sqrt{3} \cdot \frac{1}{2} \cdot \frac{1}{2} = -\frac{\sqrt{3}}{2}$

D (3) Integral pythagorean triplet must have at least one even side length (which must be a leg)

D (4) $\sin A = \frac{2x}{3}$ $\quad \therefore \cos A = \pm\sqrt{1-\frac{4x^2}{9}} = \pm\frac{\sqrt{9-4x^2}}{3}$

$\sec A = \pm\frac{3}{\sqrt{9-4x^2}}$

Since $\tan A < 0$ either $\sin A$ or $\cos A$ must be negative, $x > 0$ so $\sin A$ is positive. Therefore $\cos A$ is negative and $\sec A$ is also.

$\therefore \sec A = \frac{-3}{\sqrt{9-4x^2}}$

C (5) $64° 30' = 0.4305$
$- 64° 20' = -0.4331$
$10' = -0.0026$

$\frac{6'}{10'} = \frac{x}{-0.0026}$

$x = -0.00156$

$64° 26' = 0.4331 - 0.00156 = 0.4315$

B (6) $(r\sin 2\theta)^2 + r^2(\cos 2\theta)^2 = r^2\sin^2 2\theta + r^2\cos^2 2\theta$
$= r^2(\sin^2 2\theta + \cos^2 2\theta)$
$= r^2$

B (7) $\frac{1}{2}ab\sin C = \frac{1}{2}\cdot 12\cdot 7\cdot \frac{\sqrt{2}}{2} = 21\sqrt{2}$

C (8) Identity

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B (9) $A = \text{Arctan}\,\frac{4}{3}$ $\quad B = \text{Arccot}\,\frac{3}{4}$
$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$= \frac{4}{5}\cdot\frac{3}{5} + \frac{3}{5}\cdot\frac{4}{5}$
$= \frac{24}{25}$

A (10) $\sin x + \cos x = 1 = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$
$\frac{\sqrt{2}}{2} = \cos\left(x - \frac{\pi}{4}\right)$
$x - \frac{\pi}{4} = \frac{\pi}{4}$ $\qquad x - \frac{\pi}{4} = -\frac{\pi}{4}$
$x = \frac{\pi}{2}$ $\qquad x = 0$ $\qquad$ Sum is $\frac{\pi}{2}$

D (11) $\text{Arccos}\,2x = \text{Arcsin}\,x$
$= \text{Arccos}\sqrt{1-x^2}$
$2x = \sqrt{1-x^2}$
$4x^2 = 1 - x^2$
$5x^2 = 1$
$x^2 = \frac{\sqrt{5}}{5}$
$x = \pm\frac{\sqrt{5}}{5}$ $\quad$ negative answer will not work because of the ranges of the Arc functions
$\therefore x = \frac{\sqrt{5}}{5}$

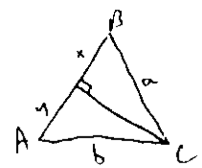
A (12) $(1-\cos x)\left(\frac{1}{\sin x} + \frac{\cos x}{\sin x}\right) = (1-\cos x)\left(\frac{1+\cos x}{\sin x}\right) = \frac{1-\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$

A (13) We need to find the sum of the roots of the equation $x^5 = 32$.
$x^5 - 32 = 0$ $\qquad \therefore$ sum $= 0$

A (14) $\sin 55° + \sin 35° = 2\sin\left(\frac{55+35}{2}\right)\cos\left(\frac{55-35}{2}\right)$
$= 2\sin 45° \cdot \cos 10°$
$= \sqrt{2}\cos 10°$

D (15) $\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$
$= (2\cos^2\theta - 1)\cos\theta - 2\sin^2\theta\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$
$= 4\cos^3\theta - 3\cos\theta$

C (16)



$x + y = c$; $x = a\cos B$; $y = b\cos A$
$\therefore a\cos B + b\cos A = c$

E (17) $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 9 + 16 - 24\cos 60°$

$c^2 = 9 + 16 - 24(\frac{1}{2})$

$c^2 = 25 - 12$

$c^2 = 13 \quad \therefore \quad c = \sqrt{13}$

B (18) Write forces in terms of vectors

$F_1: 6\cos 60° i + 6\sin 60° j = 3i + 3\sqrt{3} j$

$F_2: 9\cos 180° i + 9\sin 180° j = -9i + 0j$

$F_3: 16\cos 300° i + 16\sin 300° j = 8i - 8\sqrt{3} j$

Net force vector is just the sum of all of those vectors.

$F_N = 2i - 5\sqrt{3} j$

$Mag(F_N) = \sqrt{4 + 75} = \sqrt{79}$

C (19) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(150° + 45°) = \sin 150° \cos 45° + \cos 150° \sin 45°$

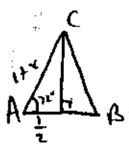
$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

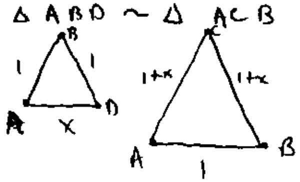
A (20) This is an expansion of $\cos x$

C (21) $\sin 18° = \cos 72°$

$\cos 72° = \frac{\frac{1}{2}}{1+x}$



$\triangle ABD \sim \triangle ACB$



Hence $\frac{1+x}{1} = \frac{1}{x}$

Solving for x: $x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{5}}{2}$

Using only positive number $x = \frac{-1+\sqrt{5}}{2}$

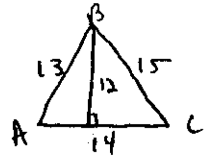
$\sin 18° = \frac{\frac{1}{2}}{1 + (\frac{-1+\sqrt{5}}{2})}$

$= \frac{1}{2 - 1 + \sqrt{5}}$

$= \frac{1}{1+\sqrt{5}}$

$= \frac{\sqrt{5}-1}{4}$

A (22) Special Triangle. Only triangle with consecutive sides and altitude.



D (23) We must reduce this into an equation similar to

$h = a\cos b(t-c) + d$

- since diameter of the wheel is 40, $a = 20$
- the max height is 45 so $d = 25$
- she makes a revolution every 10 seconds which is the period. Thus $10 = \frac{2\pi}{b}$, $b = \frac{\pi}{5}$
- At $t=4$ we have a max so that means the graph is shifted 4 units to the right $\therefore c = 4$

$h = 20\cos\frac{\pi}{5}(t-4) + 25$

C (24) Since the coefficient of θ is odd the number of leaves is just that number.

B (25) linear speed = angular velocity × radius

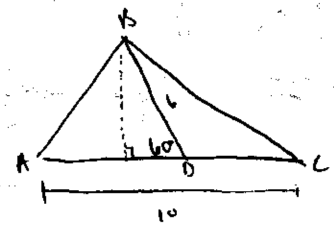
$= 3\pi \times 6$

$= 18\pi$

D (26) They intersect at $(0,0), (3/2, \frac{\pi}{6}), (\frac{3}{2}, \frac{5\pi}{6})$.

$\therefore$ answer $= 3$

D (27)



$\angle D$ must be equal to $60°$.

The altitude is equal to $6\sin 60°$, or $3\sqrt{3}$. Area is equal to $\frac{1}{2}(\text{base})\cdot(\text{height})$

$\therefore K = \frac{1}{2}\cdot(3\sqrt{3})(10) = 15\sqrt{3}$

E (28) Take $\frac{5\pi}{4}$ for x. None work

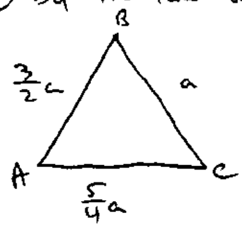
E (24) The period of $\sin \frac{2}{3}x$ is $\frac{2\pi}{\frac{2}{3}} = 3\pi$

The period of $\cos \frac{7}{8}x$ is $\frac{2\pi}{\frac{7}{8}} = \frac{16\pi}{7}$.

We must now find the least common multiple between the two (3 & $\frac{16}{7}$). Since we have an integer involved the answer must be an integer. Hence we must multiply the second one by a multiple of seven. When ~~multipling~~ multipling by 21 we get 48 which is also a multiple of 3.

Thus the period is 48π.

D (30) By the law of sines $b = \frac{5}{4}a$ and $c = \frac{3}{2}a$.



Using the law of ~~cosines~~ cosines to find the cosine of the angles:

$$a^2 = \left(\tfrac{3}{2}a\right)^2 + \left(\tfrac{5}{4}a\right)^2 - 2\left(\tfrac{3}{2}a\right)\left(\tfrac{5}{4}a\right)\cos A$$
$$\cos A = \frac{3}{4}$$

$$\left(\tfrac{5}{4}a\right)^2 = a^2 + \left(\tfrac{3}{2}a\right)^2 + 2(a)\left(\tfrac{3}{2}a\right)\cos B$$
$$\cos B = \frac{9}{16}$$

$$\left(\tfrac{3}{2}a\right)^2 = a^2 + \left(\tfrac{5}{4}a\right)^2 - 2(a)\left(\tfrac{5}{4}a\right)\cos C$$
$$\cos C = \frac{1}{8}$$

$$\cos A : \cos B : \cos C = \frac{3}{4} : \frac{9}{16} : \frac{1}{8} = 12 : 9 : 2$$

$\therefore$ $x = 12$ & $y = 9$, $x + y = 21$