State Convention
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Trigonometry

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1. Evaluate $\cos \frac{19\pi}{6}$.
   a) $-\sqrt{3}/2$
   b) $-1/2$
   c) $1/2$
   d) $\sqrt{3}/2$
   e) none of these

2. In triangle ABC, AB = $3\sqrt{3}$, BC = $5\sqrt{2}$ and AC = $\sqrt{13}$. What kind of triangle is ABC?
   a) not a triangle
   b) acute
   c) right
   d) obtuse
   e) none of these

3. In rhombus WXYZ, WY = 6 and XZ = 8. Find cos W.
   a) $-.7$
   b) $-.28$
   c) $.68$
   d) $.82$
   e) none of these

4. If $\cos 2x = 2/3$, then find $\sin^2 x$.
   a) 1/6
   b) 1/3
   c) 2/3
   d) 5/6
   e) none of these

5. Simplify the expression cos (2 Arcsin x) for permissable values of x.
   a) $2\sqrt{1 - x^2}$
   b) $\frac{\pi}{2} - x^2$
   c) $2x^2 - 1$
   d) $1 - 2x^2$
   e) none of these

6. Evaluate sec [Arctan $(2 + \sqrt{3})$ - Arctan $(2 - \sqrt{3})$].

a) $\sqrt{5} + 1$

b) $\sqrt{5} + \sqrt{2}$

c) $2\sqrt{3}/3$

d) 2

e) none of these

7. Find the total surface area of a tetrahedron with vertices at the origin, (1, 0, 0), (0, -2, 0) and (0, 0, 4).

a) $3 + 2\sqrt{2} + \sqrt{21}$

b) $7 + \sqrt{21}$

c) $7 + 2\sqrt{21}$

d) 8

e) none of these

8. What is the result of the multiplication i X k?

a) 0

b) -j

c) j

d) $\sqrt{2}$j

e) none of these

9. What is the value of $\cos^4 x - \cos 2x - \sin^4 x$ when cot x = 6/5 ?

a) $(11 - 12\sqrt{61})/61$

b) 77/324

c) 550/3721

d) 22/61

e) none of these

10. Beowulf, Dante and the Ancient Mariner each have a different favorite trig function chosen from sin x, cos x, tan x, cot x, sec x, csc x. Beowulf's function is the derivative of Dante's function, and the graph of the Mariner's function does not intersect with the graph of Dante's. Which of these functions is/are even?

a) Beowulf's

b) Dante's

c) Beowulf's and the Ancient Mariner's

d) Dante's and the Ancient Mariner's

e) none of these

11. What is the name of the conic section whose polar equation is:

$$r = \frac{1}{3 + \sin\theta}$$

a) circle
b) ellipse
c) hyperbola
d) parabola
e) none of these

12. A and B are vectors such that $|A| = 3$, $|B| = 5$ and $|A \times B| = 10$. What is the value of $|A \cdot B|$?

a) 5
b) $5\sqrt{2}$
c) $5\sqrt{5}$
d) 10
e) none of these

13. The expression $\tan(2\ \mathrm{Tan}^{-1}\ x)$ is undefined when $x =$ _____.

a) $\pm\pi/2$
b) $\pm\pi/4$
c) ± 1
d) any non-zero integer
e) none of these

14. What is the sine of the angle that the vector $6\sqrt{3}i + 5j + 6k$ makes with the y-axis?

a) 1/13
b) 5/13
c) 8/13
d) 12/13
e) none of these

15. A clown rides a unicycle at a constant rate so that the wheel's angular velocity is 20 radians per second. How long does it take the clown to travel 600 feet if the wheel's diameter is 18 inches?

a) 30 seconds
b) 40 seconds
c) 60 seconds
d) 80 seconds
e) none of these

16. What is the area of the ellipse given by the parametric equations $x = 3 + 3 \cos t$, $y = -1 + 5 \sin t$?

a) 15π

b) 16π

c) 24π

d) 36π

e) none of these

17. The expression $\cos(\tan^{-1} 2/3)$ is equal to the roots of which of the following equations?

a) $x^2 - 9/13 = 0$

b) $x^2 - 3/13 = 0$

c) $x^2 - 13/9 = 0$

d) $x^2 - 13/4 = 0$

e) none of these

18. What is the tangent of the acute angle formed by the asymptotes of $25x^2 - 36y^2 = 900$?

a) 5/3

b) 12/5

c) 60/11

d) 60/61

e) none of these

19. Solve the equation $2\cos^2 x + 5 \sin x = 4$ if $x \in [0, 2\pi)$.

a) $\pi/6, 11\pi/6$

b) $\pi/3, 5\pi/3$

c) $\pi/6, 5\pi/6$

d) $\pi/3, 2\pi/3$

e) none of these

20. To eliminate the xy term from $5x^2 + 4xy + 2y^2 + 40 = 0$, we can rotate the x and y axes counterclockwise by an acute angle θ. What is $\tan \theta$?

a) 1/3

b) 1/2

c) 3/4

d) 4/3

e) none of these

21. A, B, C and D are vertices of a regular tetrahedron. M is the midpoint of BC, and N is the midpoint of CD. What is the value of cos A in triangle MAN?

a) 1/3
b) 1/2
c) 2/3
d) 5/6
e) none of these

22. If θ is the angle between the vectors $3i - 2j + k$ and $5i - 4j - k$, find $\tan \theta$.

a) $\sqrt{11}/11$
b) $\sqrt{26}/11$
c) $7\sqrt{3}/11$
d) $7\sqrt{13}/11$
e) none of these

23. ABCD is an isosceles trapezoid where $\cos A \cos B = 1/9$ and $AB < CD$. If $AB = 3$ and $BD = 4$, then find AD.

a) $\sqrt{2}$
b) $\sqrt{2} + 1$
c) $\sqrt{17}$
d) $2\sqrt{2} - 1$
e) none of these

24. If $3 \cos \theta + 4 \sin \theta$ is expressed in the form $k \cos (\theta + \alpha)$, then what is the value of $\sin \alpha$?

a) 4/5
b) 3/5
c) -3/5
d) -4/5
e) none of these

25. What is the period of the graph of $y = |\sin 3x|$?

a) $\pi/6$
b) $\pi/5$
c) $\pi/3$
d) $2\pi/3$
e) none of these

26. What is the area of the projection of $x^2 + y^2 - 4x + 8y - 16 = 0$ onto the xy plane after it has been rotated 60° out of the plane?

a) $9\sqrt{2}\,\pi$

b) $9\sqrt{3}\,\pi$

c) $18\,\pi$

d) $18\sqrt{3}\,\pi$

e) none of these

27. Evaluate : $\dfrac{(2 \text{ cis } 20°)^6 \, (3 \text{ cis } 10°)^3}{(\text{cis } 15°)^4 \, (\cos 30°)^4}$.

a) 1728

b) 3072i

c) $864\sqrt{3} - 864i$

d) $6\sqrt{3} + 6i$

e) none of these

28. Which of the following expressions has the largest value? (argument in radians)

a) sin 1

b) cos 1

c) tan 1

d) cot 1

e) none of these

29. In triangle ABC, AC = 6, BC = 10 and cos C = 4/5. What is the length of the altitude to side $\overline{AC}$?

a) 3

b) 4

c) 6

d) 8

e) none of these

30. Find the value of A - B if $A = \cot^2 10° + \cot^2 20° + \cot^2 30° + \cot^2 40°$, and $B = \sec^2 50° + \sec^2 60° + \sec^2 70° + \sec^2 80°$.

a) -4

b) 4

c) 16

d) $\tan^2 100°$

e) none of these