13

Precalculus Individual Test

Plant

March Regional '99

The abbreviation NOTA denotes "None of These Answers."

1. $Arc\cot(-1) =$

A) $\frac{\pi}{4}$ B) $\frac{-\pi}{4}$ C) $\frac{3\pi}{4}$ D) $\frac{-3\pi}{4}$
E) NOTA

2. $\lim_{t\to 1} \frac{t^3-1}{t-1} =$

A) 0 B) 1 C) 2 D) does not exist
E) NOTA

3. The arithmetic mean of two numbers is 8 and the geometric mean of the same two numbers is 4. Which is the sum of the squares of the two numbers?

A) 0 B) 224 C) 240 D) 256
E) NOTA

4. A plane flies at a height of 300 feet above ground and passes over point P on the ground. At the same time, the plane sights point Q on the ground, with an angle of depression of 38 degrees from the plane. If the ground is perfectly level, to the nearest tenth of a foot, how far is point P from point Q?

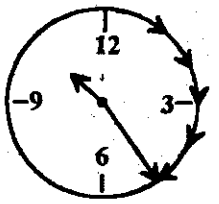
A) 184.7 ft B) 234.4 ft
C) 384.0 ft D) 487.3 ft E) NOTA

5. Which is a point of trisection of $\overline{RS}$ if the coordinates of the endpoints of $\overline{RS}$ are R$(-1, 8)$ and S$(11, -7)$?

A) $(4, 1\frac{3}{4})$ B) $(7, -2)$
C) $(5, 3)$ D) $(3, 2)$ E) NOTA

6. The minute hand of a clock is 3.5 inches long. Through what distance in inches does the tip of the minute hand travel in 25 minutes?

A) $\frac{35\pi}{12}$ B) $\frac{5\pi}{12}$
C) $\frac{245\pi}{48}$ D) $\frac{12\pi}{5}$
E) NOTA



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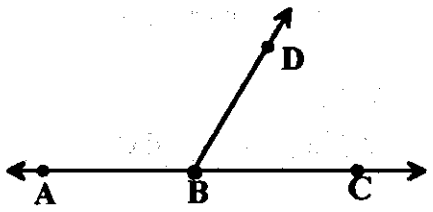
7. In ΔABC, AB=12, AC=4 and BC=10. What is the value of cos(A) ?

A) $\frac{-5}{8}$ B) $\frac{-5}{16}$ C) $\frac{5}{8}$ D) $\frac{5}{16}$ E) NOTA

8. Angles ABD and DBC form a linear pair, as shown. If $\cos(\angle ABD) = \frac{-1}{3}$, then $\sin(\angle DBC) =$

A) $\frac{3-\sqrt{2}}{3}$
B) $\frac{2\sqrt{2}-3}{3}$
C) $\frac{-2\sqrt{2}}{3}$
D) $\frac{2\sqrt{2}}{3}$
E) NOTA



9. If $Arc\cos(a+b) = \frac{\pi}{2}$ and a and b are nonzero real numbers, then which is equal to the expression $|a| + |b|$?

A) 2|a| B) 0 C) 1 D) 2 E) NOTA

10. A circle C contains the points (3, 4) and (7, 0). Which point could <u>not</u> be the center of circle C?

A) $(-3, -6)$ B) $(0, -3)$
C) $(-4, 1)$ D) $(10, 7)$ E) NOTA

11. Let $f(x) = \sin(x)$. Which is an expression for $f(x+1)$?

A) $\sin(x)+1$
B) $\sin(x)\sin 1 + \cos(x)\cos 1$
C) $\sin(x)\cos(1) + \cos(x)\sin(1)$
D) $\sqrt{1-\cos(x)}$
E) NOTA

12. $\lim_{x\to 4} f(x) = m$ and $\lim_{x\to m} f(x) = n$.
The function f and its inverse $f^{-1}(x)$ both have a domain of all Real Numbers. Find $\lim_{x\to n} f^{-1}(x)$.

A) m B) n C) 4 D) $\frac{1}{4}$ E) NOTA

13. A closed box (rectangular prism) has a surface area of 100 square feet and a square base with each base edge x feet. Give the volume of the box as a function of x, for V measured in cubic feet and x in feet.

A) $V(x) = (6x)^2 - \frac{1}{2}x$
B) $V(x) = 6x^2 + 4x$
C) $V(x) = \frac{50}{3}x - x^3$
D) $V(x) = 25x - \frac{1}{2}x^3$
E) NOTA

14. An elliptical disk of negligible thickness, in plane H, is rotated 360 degrees clockwise in H. The center of rotation is the ellipse's center. The equation which determines the perimeter of the disk is $x^2 + 2y^2 + 4x - 16y - 2 = 0$. Find the total area covered in plane H by the rotation of the disk.

A) $19\pi\sqrt{2}$ B) 22π
C) 19π D) 38π E) NOTA

15. A square of side length 10 units is in the xy-plane with the intersection of the diagonals of the square at the origin. The line $y = x$ contains one of the square's diagonals. The solution to the system $\{(x,y) : x - 3y \geq 0 \text{ and } x + 2y \leq 5\}$ is shaded on that same plane. Find the area of the shaded region that is contained within the square region.

A) $48.\overline{3}$ B) 2.5
C) $31.\overline{6}$ D) $63.\overline{3}$ E) NOTA

16. Which of the following contains the point P$(2, -\pi)$, for P written in polar form ?

A) $r = \cos(2\theta)$ B) $r = 2\cos\theta$
C) $r = -\cos(-2\theta)$ D) $r = -2\cos\theta$
E) NOTA

17. Convert the equation $x^2 = 9 - y^2$, written in rectangular form, to polar form.

A) $r = 3$ B) $r = 3\cos\theta$
C) $r = 3\sin\theta$ D) $r = \cos(2\theta)$
E) NOTA

18. For $f(x) = \begin{cases} \sin(2x) & \text{for } x \geq 1 \\ \cos(5x) & \text{for } x < 1 \end{cases}$
give the value of $f\left(\frac{\pi}{6}\right)$.

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{2}$ C) $\frac{-\sqrt{3}}{2}$ D) $\frac{-1}{2}$
E) NOTA

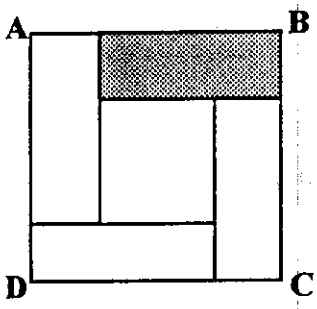
19. For $g(x) = \cos(4x) - 1$ find the distance between the points $(\pi, g(\pi))$ and $(3\pi, g(3\pi))$.

A) $2\sqrt{1+\pi^2}$ B) 2π
C) $2\pi\sqrt{2}$ D) $2 + 2\pi$ E) NOTA

20. The circle defined by $x^2+y^2+ky+c=0$ has center (0, 4) and radius 5. Give the value of $(c+k)$.

A) −33 B) −17 C) 8 D) 29 E) NOTA

21. The area of square ABCD shown consists of that of four smaller congruent rectangles and a smaller square. Each rectangle (one is shaded) has sides of lengths $\cos(x)$ and $\sin(x)$, for some angle of measure x such that $0<x<\frac{\pi}{2}$. The area of square ABCD is $\frac{7}{5}$. Give the value of $\sin(2x)$.



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A) $\frac{2}{5}$ B) $\frac{3}{5}$ C) $\frac{24}{25}$ D) $\frac{2\sqrt{6}}{5}$ E) NOTA

22. A relation f is defined as $f(x,y)$ equals the base y representation of a number N, when x is the base ten representation of N. If $f(x,5)=101$ then x =

A) 6 B) 26 C) 401 D) 505 E) NOTA

23. For $f(x+1)=x^3-2x^2+3x+8$ give the remainder when f(x) is divided by the binomial $(x-5)$.

A) 170 B) 98 C) 52 D) 3 E) NOTA

24. If $\cos x+\cos^2 x+\cos^3 x+\ldots=2$ for $0<x<\frac{\pi}{2}$ then $\sin x=$

A) $\frac{2}{3}$ B) $\frac{1}{3}$ C) $\frac{\sqrt{5}}{3}$ D) $\frac{2\sqrt{5}}{5}$ E) NOTA

25. Let a function f(x) be defined as $f(x)=C(x,8)$ for $x\geq 8, x\in Integers$ and let C(n,r) denote the number of combinations possible of n objects taken r at a time. If $f(k)=f(k-1)+330$ then find the positive integer value k for which this is true.

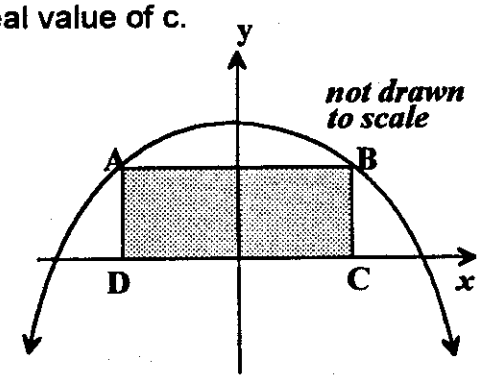
A) 11 B) 12 C) 33 D) 39 E) NOTA

26. In triangle ADC, angle D has a tangent of $\frac{1}{4}$. AD= $4\sqrt{17}$ cm and DC=10 cm. Find the length AC to the nearest hundredth of a cm.

A) 7.21 B) 8.53 C) 26.31 D) 14.04 E) NOTA

27. A parabola defined by $y=c-x^2$ for $c>0$ contains two of the vertices of rectangle ABCD as shown. One side of ABCD is on the x-axis. Rectangle ABCD has perimeter 20 units and its sides are of positive integral lengths. Give the largest possible real value of c.

A) 18.5
B) 22
C) 21.25
D) 25
E) NOTA



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28. If $12.5\sin(8x)\cos(8x)=k\sin(16x)$ then give the value of $\tan(k\pi)+\sin(k\pi)$.

A) $1+\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{3}+\sqrt{2}}{3}$

C) $\frac{3}{2}$ D) $1+\frac{\sqrt{3}}{2}$

E) NOTA

29. Five fair two-sided coins are tossed, and all land showing either heads or tails. If it is known that three of the five showed heads, what is the probability that all of the five coins showed heads?

A) $\frac{1}{32}$ B) $\frac{1}{16}$ C) $\frac{1}{8}$ D) $\frac{1}{4}$ E) NOTA

30. Let $f(x) = x^3 + 3x^2 + 3x + 1$ and $g(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$, and let $h(x) = g(x) - f(x)$.

If $h(4000) = n^3(n-1)$ then $n =$

A) 80 B) 4001
C) 4002 D) 4003 E) NOTA