P.C. CALCULUS

#12

Armwood
SOLUTIONS TO
30 QUESTION MULTIPLE CHOICE TEST.

MARCH REGIONAL
3/11/95

1) THE AREA OF A TRIANGLE CAN BE FOUND USING DETERMINANTS.

$$A = \pm\frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -2 & 5 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 18 \quad (b)$$

2) VERT-ASYM. ARE FOUND BY FACTORING THE DENOMINATOR

$x^2-4 = (x+2)(x-2)$, THE 2 VERT ASY ARE $x=2, x=-2$ (b)

3) a) $[p = b\sin A = 13]$ $p < a < b$, 2 CASES

b) FROM SIMILAR TRIANGLES, INFINITE SOLUTIONS

c) $a < p$, NO SOLUTION

(d) $a = p$, ONE UNIQUE SOLUTION

4) WHEN $u \cdot v = 0$ THEN 2 VECTORS ARE PERPENDICULAR

(c) $(-1,3), (6,2) \Rightarrow (-1,3)\cdot(6,2) = -6+6=0$ ✓

5) $(\sqrt{3}, -1) \Rightarrow 2\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right) \Rightarrow 30°$ REF ANGLE WHERE $r = 2$

$(2, 330°)$ (c)

6) [redacted]

(c) $a_1 = \frac{3}{\pi}$, $r = \frac{1}{\pi}$ $\therefore$ $\text{Sum} = \frac{a_1}{1-r}$

$$\text{Sum} = \frac{\frac{3}{\pi}}{1-\frac{1}{\pi}} \Rightarrow \frac{\frac{3}{\pi}}{\frac{\pi-1}{\pi}} \Rightarrow \frac{3}{\pi-1}$$

7) THIS IS A CONDITIONAL PROBABILITY. THE CONDITION LIMITS THE SAMPLE SET TO 12 POSSIBILITIES. "AT MOST" MEANS 6 OR LESS, THEY ARE

(1,1), (1,2) (1,3) (1,4) (1,5) (2,1) (2,2) (2,3) (2,4)

$\frac{9}{12}$ or $\frac{3}{4}$ ans. E

PRE CALCULUS — SOLUTIONS 10 — MARCH REGIONAL 3/11/95

30 QUESTION MULTIPLE CHOICE TEST

8) SLOPE IS DEFINED AS $\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ so $\lim\limits_{h \to 0} \dfrac{[1-(x+h)^2]-(1-x^2)}{h} \Rightarrow$

$$\lim_{h \to 0} \frac{(1-x^2-2xh-h^2)-1+x^2}{h} \Rightarrow \lim_{h \to 0} \frac{-2xh-h^2}{h} \Rightarrow \lim_{h \to 0} -2x-h \Rightarrow$$

$= -2x$ ∴ EVALUATE AT $x=-2$ ∴ $m = (-2x) \Rightarrow 4$

PLUG IN $(-2,-3)$ TO FIND b FOR $y = mx+b \Rightarrow -3 = 4(-2)+b \Rightarrow b=5$

∴ THE EQUATION OF THE TANGENT LINE IS $y = 4x+5$ (a.)

9) $\lim\limits_{x \to 0} \left(\frac{1}{2}x - \frac{1}{3}\right) = \frac{1}{2}(0) - \frac{1}{3} = -\frac{1}{3}$ (d.)

10) $\cos 7x + \cos 3x \Rightarrow \cos(5x+2x) + \cos(5x-2x) \Rightarrow$

$(\cos 5x \cos 3x - \sin 5x \sin 3x) + (\cos 5x \cos 3x + \sin 5x \sin 3x) \Rightarrow$

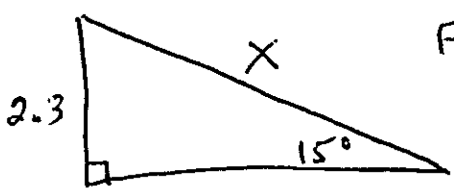
$2\cos 5x \cos 3x$ (d.)

11) PHASE SHIFT $= \pi/4$

PERIOD $= \frac{2\pi}{3}$

$\left|\frac{\pi}{4} - \frac{2\pi}{3}\right| = \left|\frac{-5\pi}{12}\right| = \frac{5\pi}{12}$ (c)

12)



FIND X $\quad \sin 15° = \frac{2.3}{x}$

$x = \frac{2.3}{\sin 15°} \approx 8.88 \Rightarrow 8.9$ (a)

13) $\left|3\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right| = \left|\frac{3\sqrt{3}}{2} - \frac{3}{2}i\right| = \sqrt{\left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{-3}{2}\right)^2} = 3$ (d)

14) $a, \left| \dfrac{a+\frac{a+b}{2}}{2}, \dfrac{a+b}{2}, \dfrac{b+\frac{a+b}{2}}{2} \right|, b \Rightarrow \dfrac{a+\frac{a+b}{2}+a+b+b+\frac{a+b}{2}}{2} \Rightarrow$

$\dfrac{2a+2b+a+b}{2} \Rightarrow \dfrac{3a+3b}{2}$ (E) NONE OF THESE

PRE CALCULUS SOLUTIONS TO MARCH REGIONAL 3/11/95

30 QUESTION MULTIPLE CHOICE TEST

15) $-2 \lim_{x \to 2^-} \left(\frac{4}{x-2}\right)$, 2 FROM THE LEFT HAND SIDE APPROACHES THE DENOMINATOR TO BECOME CLOSE TO ZERO BUT NEGATIVE. ANY CONSTANT DIVIDED BY A NUMBER APPROACHING ZERO RISES TO NEGATIVE INFINITY. BUT THE -2 OUTSIDE THE LIMIT CHANGES TO OVERALL LIMIT TO A POSITIVE INFINITY $\Rightarrow (-\infty)(-2) \Rightarrow \infty$ d.

16) a. $f(2) = 2(2)^4 - 3(2)^2 + c(2) + k = 24 \Rightarrow f(2) = 2c + k = 4$
b. $f(-2) = 2(-2)^4 - 3(-2)^2 + c(-2) + k = 20 \Rightarrow f(-2) = -2c + k = 0$

add a & b $2k = 4$, $k = 2$ PLUG K BACK IN $2c + 2 = 4$, $c = 1$

$\therefore$ $k = 2$, $c = 1$ AND $c + k = 3$ b

17) $(\log_{10} x)^2 = \log_{10} x^2 \Rightarrow (\log_{10} x)(\log_{10} x) = 2\log_{10} x$

DIVIDE BOTH SIDES BY $(\log_{10} x) \Rightarrow \log_{10} x = 2 \Rightarrow \underline{x = 10^2}$

ALSO $(\log_{10} x)$ COULD EQUAL ZERO SO $\log_{10} x = 0 \Rightarrow \underline{x = 10^0}$

$\therefore$ THE 2 SOLUTIONS ARE 100 & 1 $\therefore$ $\sqrt{100} + \sqrt{1} \Rightarrow 10 + 1 \Rightarrow 11$ c

18) $\sqrt{(0-(-\sqrt{10}))^2 + (1-(-4))^2 + (2-1)^2} = \sqrt{10 + 25 + 1} = \sqrt{36} = 6$ b

19) DIRECTION VECTOR FOR l_1 IS $\vec{u} = 2i + 5j + 0k$
" " l_2 IS $\vec{v} = 3i + j + 0k$

FORMULA $\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$

$\cos\theta = \frac{2 \cdot 3 + 5 \cdot 1 + 0 \cdot 0}{(\sqrt{29})(\sqrt{10})} \Rightarrow \cos\theta = \frac{11}{\sqrt{290}} \Rightarrow \theta \approx 49.76° \Rightarrow \theta = 50°$ c

20) $x = x' + 3$, $y = y' - 2$

BY SUBSTITUTION $(y' - 2) = 2(x' + 3) - 3 \Rightarrow y' = 2x' + 6 - 3 + 2 \Rightarrow$

$y' = 2x' + 5$ $\therefore$ $y = 2x + 5$ b

21)
$$\begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}\begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}\begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \end{bmatrix} \Rightarrow \begin{bmatrix} -2 \\ 0 \end{bmatrix} \quad \text{(c)}$$

22) FIRST COMPLETE THE SQUARE:
$$4(x^2-2x+\underline{\ 1\ })+9(y^2-6y+\underline{\ 9\ }) = -49+4+81$$
$$4(x-1)^2+9(y-3)^2 = 36$$
$$\frac{(x-1)^2}{9}+\frac{(y-3)^2}{4} = 1$$
LATUS RECTUM $\frac{2b^2}{a} \Rightarrow$
$$\frac{2(2)^2}{3} \Rightarrow \frac{8}{3} \quad \text{(a)}$$

23) THE SAMPLE CONTAINS ${}_7P_5$ ELEMENTS
THE EVENT HAS $4\times5\times4\times3\times3$ ELEMENTS
$$p = \frac{4\cdot5\cdot4\cdot3\cdot3}{7\cdot6\cdot5\cdot4\cdot3} = \frac{2}{7} \quad \text{(d.)}$$

24)
$$\sin\left(\tfrac{\pi}{4}+x\right)-\sin\left(\tfrac{\pi}{4}-x\right) = \frac{\sqrt{2}}{2} \Rightarrow \sin\tfrac{\pi}{4}\cos x+\cos\tfrac{\pi}{4}\sin x-\left(\sin\tfrac{\pi}{4}\cos x-\cos\tfrac{\pi}{4}\sin x\right) \Rightarrow$$
$$\left(\sin\tfrac{\pi}{4}\cos x-\sin\tfrac{\pi}{4}\cos x\right)+\cos\tfrac{\pi}{4}\sin x+\cos\tfrac{\pi}{4}\sin x = \frac{\sqrt{2}}{2}$$
$$0+2\cos\tfrac{\pi}{4}\sin x = \frac{\sqrt{2}}{2} \Rightarrow (2)\frac{\sqrt{2}}{2}\sin x = \frac{\sqrt{2}}{2} \Rightarrow$$
$\sin x = \frac{1}{2}$, $\therefore$ $x = 30^\circ$ or 150°, BOTH WORK $\therefore$ (E.) NOTA.

25) SINCE $3i+2j$ IS A DIRECTION VECTOR THEN THE SLOPE IS $\frac{2}{3} \Rightarrow$
$$(y-2) = \tfrac{2}{3}(x-5) \Rightarrow 3y-6 = 2x-10 \Rightarrow 2x-3y = 4 \quad \text{(b.)}$$

PRE CALCULUS

SOLUTIONS TO

30 QUESTION MULTIPLE CHOICE TEST

MARCH REGIONAL

3/11/95

26)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -4 & 2 & 16 \\ 3 & 1 & -1 & -2 \end{array}\right] \Rightarrow -2r_1 + r_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -8 & -4 & 4 \\ 3 & 1 & -1 & -2 \end{array}\right] \Rightarrow -3r_1 + r_3 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -8 & -4 & 4 \\ 0 & -5 & -10 & -20 \end{array}\right];$$

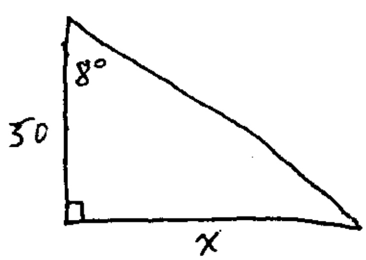
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -5 & -10 & -20 \\ 0 & -8 & -4 & 4 \end{array}\right] \Rightarrow -\frac{1}{5}r_2 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & -8 & -4 & 4 \end{array}\right] \Rightarrow 8r_2 + r_3 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 12 & 36 \end{array}\right] \Rightarrow$$

$$\frac{1}{12}r_3 \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow -2r_2 + r_1 \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow -2r_3 + r_2 \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \Rightarrow$$

$$r_3 + r_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$ c

27) FITS THE FORM OF $y = ax^2 + b$ ∴ PARABOLA d

28)



$\tan 8° = \frac{50}{x} \Rightarrow x = \frac{50}{\tan 8°} \Rightarrow x \approx 355.76$

$x \approx 356$ m b

29) $\frac{PRECALCULUS!}{(CC)!(LL)!(UU)!} \Rightarrow \frac{(11)!}{2!2!2!} \Rightarrow 4{,}989{,}600$ c

30) SINCE C REPRESENTS THE VERTICAL SHIFT AND $c = 0$

THEN $a \cdot b \cdot (0) = 0$ E. NOTA