

Pre-Calculus Regional
March 5, 1994
Individual Examination

(NOTA means none of the above)

1. The line through the point $(2, -1)$ and perpendicular to the line $5x - y = 3$ has y -intercept of
A) -11 B) $-7/5$ C) $-3/5$ D) 9 E) NOTA
2. Which of the following equations has a graph that is symmetric with respect to the origin?
A) $y = \frac{x-1}{x}$ B) $y = |x|$ C) $y = x^3 + 2x$ D) $y = \cos x$ E) NOTA
3.
$$\begin{vmatrix} 2 & 3 & 1 \\ -1 & 5 & 0 \\ 4 & -6 & 1 \end{vmatrix} =$$

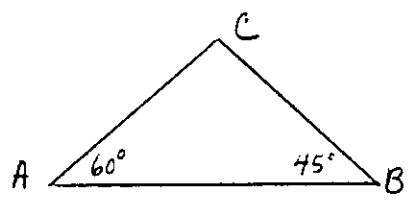
A) 1 B) 13 C) 23 D) 33 E) NOTA
4. Find the distance (to the nearest hundredth) from the line whose equation is $4x + 6y = 36$ to the point $(-2, -7)$.
A) 9.98 B) 10.53 C) 11.17 D) 11.93 E) NOTA
5. If $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$, find the inverse of A , A^{-1} .
A) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ B) $\begin{bmatrix} .4 & .2 \\ .2 & .6 \end{bmatrix}$ C) $\begin{bmatrix} .4 & -.2 \\ -.2 & .6 \end{bmatrix}$
D) $\begin{bmatrix} 2/7 & 1/7 \\ 1/7 & 3/7 \end{bmatrix}$ E) NOTA
6. The area enclosed by the ellipse $x^2 + 9y^2 = 36$ is
A) 36π B) 25π C) 18π D) 12π E) NOTA
7. Let θ be the angle formed by the diagonal of the face of a cube and the diagonal of the cube, drawn from the same vertex. Find θ to the nearest tenth of a degree.
A) 30.0° B) 35.3° C) 45.0° D) 54.7° E) NOTA
8. The distance between the two vertices (major axis) of $9x^2 + 4y^2 + 90x - 8y + 85 = 0$ is
A) 12 B) 8 C) 6 D) 4 E) NOTA

9. If $f(x) = x^3 - 3x^2 - 2x + 5$ and $g(x) = 2$, then $g(f(x)) =$
 A) $2x^3 - 6x^2 - 2x + 10$ B) $2x^2 - 6x + 1$ C) 2 D) -6 E) NOTA
10. The rational function $f(x) = \frac{x^2 - 1}{x^2 - 4x}$ crosses the
 x-axis at $x = a$ and $x = b$. The value of $a + b$ is
 A) 0 B) -2 C) 2 D) 4 E) NOTA
11. Given $8x^2 - 12xy - 8y^2 + 6\sqrt{10}x - 2\sqrt{10}y = 30$, find the
 measure of the **acute angle of rotation** (to the nearest
 tenth of a degree) that will eliminate the xy -term.
 A) 18.4° B) 36.9° C) 45.0° D) 71.6° E) NOTA
12. The roots of $x^3 + 2px^2 - px + 10 = 0$ are integral and form
 an arithmetic progression. Find the value of p .
 A) 3 B) -3 C) 6 D) -6 E) NOTA
13. If $(\log_k x)(\log_5 k) = 3$ and $k > 0$ and $x > 0$ and $k \neq 1$,
 then $x =$
 A) 125 B) $2k/125$ C) k^2 D) $5k^2$ E) NOTA
14. Find the term free of x (the constant term) in
 the expansion of $\left[2x^2 + \frac{1}{x}\right]^9$.
 A) 484 B) 672 C) 682 D) 778 E) NOTA
15. Find the value of the continued fraction $\frac{1}{2 + \frac{1}{2 + \dots}}$.
 A) $1 + \sqrt{2}$ B) $1 - \sqrt{2}$ C) $-1 + \sqrt{2}$ D) $-1 - \sqrt{2}$ E) NOTA

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16. Find the **cube root** of $27(\cos 30^\circ + i \sin 30^\circ)$ which, when represented graphically, lies in the **second quadrant**.
 A) $3(\cos 100^\circ + i \sin 100^\circ)$ B) $3(\cos 130^\circ + i \sin 130^\circ)$
 C) $3(\cos 150^\circ + i \sin 150^\circ)$ D) $3(\cos 170^\circ + i \sin 170^\circ)$ E) NOTA

17. In triangle ABC, $A = 60^\circ$ and $B = 45^\circ$. What is the **ratio** of side BC to side AC?



- A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{4}$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{\sqrt{6}}{2}$ E) NOTA

18. Find the **acute angle** of intersection (to the nearest thousandth of a degree) from a line with slope $2/3$ to a line with slope $5/3$.

- A) .442 B) 25.346 C) 54.866 D) 64.654 E) NOTA

19. The rectangular equation $x^2 + 10y + y^2 = 0$ changed into polar coordinate form is

- A) $r = 10 \sin^2 \theta$ B) $r = -10 \sin \theta$
 C) $r = 10 \cos \theta$ D) $r = -5 \sin 2\theta$ E) NOTA

20. $\lim_{h \rightarrow 0} \frac{\sqrt{(x+h)} - \sqrt{x}}{h} = ?$ for $x > 0$

- A) \sqrt{x} B) $\frac{1}{\sqrt{x}}$ C) $\frac{\sqrt{x}}{2}$ D) $\frac{1}{2\sqrt{x}}$ E) NOTA

21. If $f(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$, for what value of k

is f continuous at $x = 2$?

- A) 16 B) 12 C) 8 D) 4 E) NOTA

22. A ship sailed 42 kilometers on course 31° and then 35 kilometers on course 126° . The ship returned to its starting point by the **shorest route**. To the nearest kilometer, find the **total distance** traveled by the ship.

- A) 129 B) 130 C) 139 D) 146 E) NOTA

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23. How many points of intersection do $r = 5$ and $r = 5 \cos 4\theta$ have?

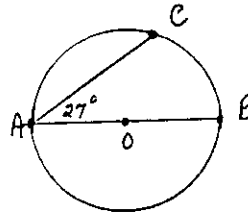
- A) none B) 4 C) 5 D) 8 E) NOTA

24. If $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$

and $x > 0$, $x \neq 1$, $x \neq 1/2$, $y > 0$, find y .

- A) 3 B) 6 C) 9 D) All positive reals E) NOTA

25. In the adjoining diagram, \overline{AB} is a diameter of circle O , making an angle of 27° with chord \overline{AC} . If \overline{AC} is 4 inches long, the length of the radius (in inches) of the circle is



- A) $2 \csc 27^\circ$ B) $4 \cos 27^\circ$ C) $2 \sec 27^\circ$ D) $4 \sin 27^\circ$ E) NOTA

26. If $y = f(x) = \frac{1-x}{1+x}$, then which one of the following is true? ($x \neq \pm 1$, $x \neq -\frac{1}{2}$, $x \neq 0$)

- A) $f(2x) = f(x) - 1$ B) $x = \frac{1}{f(y)}$ C) $x = f(2y) - 1$
 D) $x = f(y)$ E) NOTA

27. If $2^a = 4^b$ and $27^{(a-2)} = 3^{(b+1)}$, find $a + b$.

- A) 4.2 B) 2.8 C) 1.8 D) 1.4 E) NOTA

28. If u and v are the roots of $2x^2 - 5x + 1 = 0$, what is the value of $\log_2 u + \log_2 v$?

- A) -1 B) 0 C) $1/2$ D) 1 E) NOTA

29. What is the final digit (i.e., units digit) of 2^{4025} ?

- A) 2 B) 4 C) 6 D) 8 E) NOTA

30. For the function $f(x) = 2x^3 - 2x^2 + x + 8$, find the greatest negative integral lower bound for the real roots.

- A) -1 B) -2 C) -3 D) -4 E) NOTA