March 14. 1992 — PRE-CALCULUS

REGIONAL COMPETITION

ANSWERS TO INDIVIDUAL

1.	A	16.	E
2.	D	17.	A
3.	B	18.	C
4.	C	19.	D
5.	D	20.	A
6.	D	21.	E
7.	B	22.	B
8.	C	23.	E
9.	E	24.	C
10.	D	25.	B
11.	D	26.	D
12.	A	27.	C
13.	B	28.	B
14.	A	29.	E
15.	B	30.	A

ANSWERS TO TEAM

1. 5/6
2. -9
3. -3, 0, 1
4. $\dfrac{\sqrt{261}}{261}(8, \ 14, \ 1)$
5. $\dfrac{1}{1 - \cot x}$
6. $\sqrt{29}$
7. 81/80
8. 5a + -9b
9. 37
10. $y = \dfrac{4}{3}x + \dfrac{-25}{3}$ and $y = \dfrac{-4}{3}x + \dfrac{7}{3}$
11. 3/2
12. 436
13. $0 \le$ integers ≤ 51 or $0 \le$ integers < 52
14. 3
15. 2

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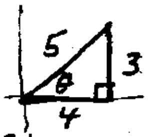
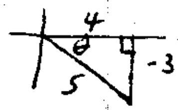
REGIONAL COMPETITION ANSWERS

INDIVIDUAL

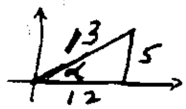
1\. $\boxed{a}$ Arcsin $\frac{3}{5}$ $\rightarrow$ 

So $\tan\left(\text{Arcsin}\frac{3}{5}\right) = \tan\theta = \frac{3}{4}$

2\. $\boxed{D}$ arctan $-\frac{3}{4}$ $\rightarrow$ 

$\sin\theta = -\frac{3}{5}$

$\cos\theta = \frac{4}{5}$

Arcsin $\frac{5}{13}$ $\longrightarrow$ 

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

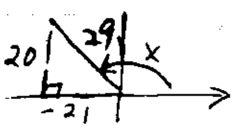
$\cos\left(\text{Arctan}\left(-\frac{3}{4}\right) + \text{Arcsin}\frac{5}{13}\right) =$

$\cos(\theta + \alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$

$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right)$

$= \frac{48}{65} - \frac{-15}{65} = \frac{63}{65}$

3\. $\boxed{B}$ Since one die is given as being a 5, there are only 11 case and 2 would give 8. So Probability is $\frac{2}{11}$

4\. $\boxed{C}$ 

$\sin x = \frac{20}{29}$ $\cos x = \frac{-21}{29}$

$\sin(x + 45^\circ) = \sin x\cos 45^\circ + \cos x\sin 45^\circ$

$= \left(\frac{20}{29}\right)\left(\frac{\sqrt{2}}{2}\right) + \frac{-21}{29}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{2}}{58}$

5\. $\boxed{D}$ $\left(e^{\ln(x-1)}\right)\left(\ln e^{x+5}\right) = 3\ln e + e^{2\ln 2}$

$(x-1)(x+5) = 3 + 4$

$x^2 + 4x - 5 = 7$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6$ and $x = 2$

6\. $\boxed{D}$ $-3 < 2x - 1 < 7$

$-2 < 2x < 8$

$-1 < x < 4$ = Domain of $h(x)$

$-2 \le f(2x-1) \le 16$

$0 \le f^2(2x-1) \le 256$

$3 \le f^2(2x-1) + 3 \le 259$

$3 \le$ Reals ≤ 259 = Range of $h(x)$

7\. $\boxed{B}$ $f(8) = f(6) + 40 = 87$

$f(6) = f(4) + 30 = 47$

$f(4) = f(2) + 20 = -3 + 20 = 17$

8\. $\boxed{C}$ $D = \frac{6(-1) + 8(5) + -14}{\sqrt{36+64}} = \frac{20}{10}$

$D = 2$ which is radius

$(x+1)^2 + (y-5)^2 = 4$

9\. $\boxed{E}$ $6x + 11y = -1 \longrightarrow 6x + 11y = -1$

$3x + y = 1 \xrightarrow{M.-2} -6x + -2y = -2$

$9y = -3$

$\boxed{y = -\frac{1}{3}}$

$3x + -\frac{1}{3} = 1$

$3x = \frac{4}{3}$

$\boxed{x = \frac{4}{9}}$ line ~~meet~~ at $\left(\frac{4}{9}, -\frac{1}{3}\right)$

$D = \sqrt{\left(\frac{4}{9} - 0\right)^2 + \left(-\frac{1}{3} - 0\right)^2} = \sqrt{\frac{16}{81} + \frac{1}{9}} = \sqrt{\frac{16}{81} + \frac{9}{81}}$

$= \sqrt{\frac{25}{81}} = \frac{5}{9}$

10\. $\boxed{D}$ $e^{\ln 7} + \ln e^{x+3} - \ln e^{x-5}$

$7 + (x+3) - (x-5)$

$7 + x + 3 + -x + 5 = 15$

11. Find cross-product of given vectors.

D

$$\begin{vmatrix} i & j & K \\ -2 & 3 & 1 \\ 4 & 1 & -3 \end{vmatrix} = i\begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} - j\begin{vmatrix} -2 & 1 \\ 4 & -3 \end{vmatrix} + K\begin{vmatrix} -2 & 3 \\ 4 & 1 \end{vmatrix}$$

$$i(^-9 + ^-1) - j(^+6 + ^-4) + K(^-2 + ^-12)$$

$$= ^-10i + ^-2j + ^-14K$$

Since $(-10,-2,-14) = -2(5,1,7)$

12. $\sin x = \frac{-5}{13}$ mean x is in Quad. III or IV

A

$\sec x = \frac{13}{12}$ means x is in Quad. I or IV

So x is in Quadrant IV.

So $x + \frac{\pi}{2}$ would be in Quadrant I

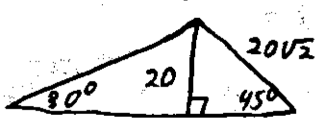
13. $12x + 18x + 42x = 180$

B

$72x = 180$

$x = \frac{5}{2}$

So angles are $30°$, $45°$ and $105°$



$\boxed{\text{altitude} = 20}$

14.

A

$$A = \frac{1}{2}\begin{vmatrix} 3 & 6 & 1 \\ 6 & 10 & 1 \\ 12 & 2 & 1 \end{vmatrix} = \frac{1}{2}\left[3\begin{vmatrix} 10 & 1 \\ 2 & 1 \end{vmatrix} + ^-6\begin{vmatrix} 6 & 1 \\ 12 & 1 \end{vmatrix} + 1\begin{vmatrix} 6 & 10 \\ 12 & 2 \end{vmatrix}\right]$$

$$= \frac{1}{2}\left[3(8) + ^-6(-6) + 1(-108)\right] = \frac{1}{2}(-48) = ^-24$$

So $A = 24$ (area ≥ 0)

$(3,6)$ To $(6,10) = \sqrt{(3-6)^2 + (6-10)^2} = \sqrt{9+16} = 5$

$(6,10)$ To $(12,2) = \sqrt{(6-12)^2 + (10-2)^2} = \sqrt{36+64} = 10$

$(3,6)$ To $(12,2) = \sqrt{(3-12)^2 + (6-2)^2} = \sqrt{81+16} = \sqrt{97}$

Since $9 < \sqrt{97} < 10$, then $24 < \text{Perimeter} < 25$

So $\boxed{48 < \text{Area} + \text{Perimeter} < 49}$

15. **B**

$\log_b \sin x = a \longrightarrow \sin x = b^a$

Since $\cos^2 x = 1 - \sin^2 x \rightarrow \cos x = \sqrt{1 - \sin^2 x}$

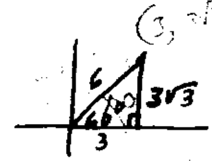
So $\log_b \cos x = \frac{1}{2}\log_b(1 - \sin^2 x)$

and $\sin^2 x = (b^a)^2 = b^{2a}$

$\boxed{\text{So } \log_b \cos x = \frac{1}{2}\log_b(1 - b^{2a})}$

16. **E** $3 + 3\sqrt{3}\,i$

So $3 + 3\sqrt{3}\,i = 6 \operatorname{Cis} 60°$



17.

A

$$\begin{vmatrix} \sin\theta & -\sin\theta & (1-\sin\theta) \\ \sin\theta & 1 & \cos\theta \\ -1 & 1 & 1 \end{vmatrix}$$

$$\sin\theta\begin{vmatrix} 1 & \cos\theta \\ 1 & 1 \end{vmatrix} - ^-\sin\theta\begin{vmatrix} \sin\theta & \cos\theta \\ -1 & 1 \end{vmatrix} + (1-\sin\theta)\begin{vmatrix} \sin\theta & 1 \\ -1 & 1 \end{vmatrix}$$

$$\sin\theta(1-\cos\theta) + \sin\theta(\sin + \cos\theta) + (1-\sin\theta)(\sin\theta + 1)$$

$$\sin\theta - \sin\theta\cos\theta + \cancel{\sin^2\theta} + \sin\theta\cos\theta + 1 - \cancel{\sin^2\theta}$$

$1 + \sin\theta$

18.

C

$$\frac{(1+i)^6(4+2i)}{\frac{1}{4}(\sqrt{2}+i\sqrt{2})^6} = \frac{\cancel{(1+i)^6}(4+2i)}{\frac{1}{4}(\sqrt{2})^6\cancel{(1+i)^6}}$$

$\frac{1}{4}(8)$

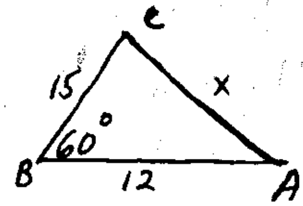
$$\frac{4+2i}{2} = \boxed{2+i}$$

19. dot product of $(4,-6)$ and $(12,8) = 48 + ^-48 = 0$

D

20.

A



Law of Cosines

$X^2 = 12^2 + 15^2 - 2(12)(15)\cos 60$, $\cos 60 = \frac{1}{2}$

$X^2 = 144 + 225 - 180 = 189$

$X = \sqrt{189} = \sqrt{9}\sqrt{21} = 3\sqrt{21}$

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21. $\boxed{E}$

$$\frac{\sec^2 x - (\tan^2 x - \sin^2 x - \cos^2 x)}{\sin^2 x(\tan x) + \tan x + (\tan x)(\cos^2 x)}$$

$$\frac{\underbrace{\sec^2 x - \tan^2 x}_{=1} + \underbrace{\sin^2 x + \cos^2 x}_{=1}}{\tan x(\sin^2 x + 1 + \cos^2 x)} = \frac{\cancel{2}\,1}{\cancel{2}\tan x}$$

22. $\boxed{B}$

$\sin 30^\circ + \cos 60^\circ + \sin 90^\circ + \cos 120^\circ + \sin 150^\circ + \cos 180 + \sin 210^\circ + \cos 240 + \sin 270^\circ + \cos 300^\circ + \sin 330^\circ + \cos 360^\circ =$

$\frac{1}{2} + \frac{1}{2} + 1 + -\frac{1}{2} + \frac{1}{2} + -1 + -\frac{1}{2} + -\frac{1}{2} + -1 + \frac{1}{2} + -\frac{1}{2} + 1 = 0$

So $\sin 390^\circ + \cos 420^\circ + \cdots + \cos 720^\circ = 0$

$\sin 750^\circ + \cos 780^\circ = \frac{1}{2} + \frac{1}{2} = \boxed{1}$

23. $\boxed{E}$

$$\begin{array}{rl} & a = 2+5+8+\cdots+896+899 \\ - & b = -(1+4+7+\cdots+895) \\ \hline & \underbrace{1+1+1+\cdots+1}_{298} \quad +899 \end{array}$$

$896 \div 3 = 298\ R2$

$899 + 298 = 1197$

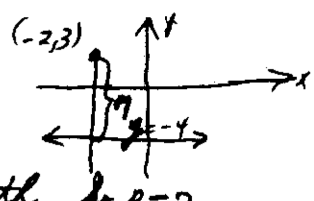
24. $\boxed{C}$

$(\sin x)(\cot x)(\sec^2 x)(\cos x)(\csc x)$

$\left(\frac{\cancel{\sin x}}{1}\right)\left(\frac{\cancel{\cos x}}{\cancel{\sin x}}\right)\left(\frac{1}{\cancel{\cos^2 x}}\right)\left(\frac{\cancel{\cos x}}{1}\right)\left(\frac{1}{\sin x}\right) = \csc x$

25. $\boxed{B}$

$(x+2)^2 + (y-3)^2 = 49$



$C(-2,3)$ with ~~So~~ $R = 7$

26. $\boxed{D}$

$(x \sin 60)^2 - x \cos 60^\circ = .5 \sin 150^\circ$

$\left(\frac{\sqrt{3}}{2}x\right)^2 - \frac{1}{2}x = \frac{1}{2}\left(\frac{1}{2}\right)$

$\frac{3}{4}x^2 - \frac{1}{2}x = \frac{1}{4}$

$3x^2 - 2x = 1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3}$ $\quad x = 1$

27. $\boxed{C}$

$x^2 + 6x + 9 = -20y + -49 + 9$ add 9 to complete sq.

$(x+3)^2 = -20(y+2)$

Vertex at $(-3,-2)$ and opens down

Focus at $(-3,-7)$

28. $\boxed{B}$

$\sin^3 x + \sin 2x(\cos x) - \frac{\cos^2 x}{\csc x}$

$\sin^3 x + 2\sin x \cos x(\cos x) - \sin x \cos^2 x$

$\sin^3 x + \sin x \cos^2 x$

$\sin x(\sin^2 x + \cos^2 x) = \sin x$

29. $\boxed{E}$

$\sin^2 x(\cos x) - \cos^3 x = 0$

$\cos x(\sin^2 x - \cos^2 x) = 0$

$\cos x(\sin x + \cos x)(\sin x - \cos x) = 0$

$\cos x = 0$	$\sin x = -\cos x$	$\sin x = \cos x$
90°, ~~270°~~	135°, ~~315°~~	45°, ~~225°~~
90°	135°	45°

$\{45^\circ, 90^\circ, 135^\circ\}$

30. $\boxed{a}$

$A_0 = \frac{1}{2}\begin{vmatrix} 3 & 2 & 1 \\ 5 & 7 & 1 \\ 11 & 2 & 1 \end{vmatrix} = \frac{1}{2}\left(3\begin{vmatrix} 7 & 1 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 5 & 1 \\ 11 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & 7 \\ 11 & 2 \end{vmatrix}\right)$

$15 + 12 + -67$

$= \frac{1}{2}\left(3(5) + -2(-6) + 1(-67)\right) = \frac{1}{2}(-40) = -20$

$A_{old} = 20$ $\quad$ area ≥ 0

$A_N = \frac{1}{2}\begin{vmatrix} -8 & 18 & 1 \\ -17 & 41 & 1 \\ -24 & 50 & 1 \end{vmatrix} = \frac{1}{2}\left(-8\begin{vmatrix} 41 & 1 \\ 50 & 1 \end{vmatrix} - 18\begin{vmatrix} -17 & 1 \\ -24 & 1 \end{vmatrix} + 1\begin{vmatrix} -17 & 41 \\ -24 & 50 \end{vmatrix}\right)$

$= \frac{1}{2}\left(-8(-9) + -18(7) + 1(134)\right) = \frac{1}{2}(80) = 40$

So diff. of areas $= 20$