

Armwood '96

Geometry Team Questions March Solutions

Team Question #1: 165

let x = angle

$90 - x$ = complement

$180 - x$ = supplement

$$180 - x = 3(90 - x) - 60 \rightarrow 180 - x = 270 - 3x - 60 \rightarrow x = 15$$

$$\text{supplement} = 180 - 15 = 165$$

Team Question #2: 126

$$2x^2 - 5 = 5x + 7 \rightarrow 2x^2 - 5x - 12 = 0 \rightarrow (2x + 3)(x - 4) = 0 \rightarrow x = -3/2 \text{ (reject) or } x = 4$$

$$\text{base angle} = 5(4) + 7 = 27, \text{ vertex angle} = 180 - (27 + 27) = 126$$

Team Question #3: Isosceles and Right

$$-- = \sqrt{(-3-4)^2 + (1-2)^2} = \sqrt{50}$$

$$FG = \sqrt{(4-1)^2 + (2-(-2))^2} = \sqrt{25} = 5$$

$$GE = \sqrt{(-3-1)^2 + (1-(-2))^2} = \sqrt{25} = 5 \quad \text{Isosceles}$$

$$5^2 + 5^2 = \sqrt{50^2}$$

$$50 = 50 \quad \text{Right}$$

Team Question #4: 56

8 rays form 28 angles, so 8 distinct lines passing through a point will create twice as many pairs of vertical angles

Team Question #5: $3x - 8y = -12$

$$5(8x + 3y = 41) \rightarrow 40x + 15y = 205$$

$$\underline{-3(6x + 5y = 39)} \rightarrow \underline{-18x - 15y = -117}$$

$$22x = 88$$

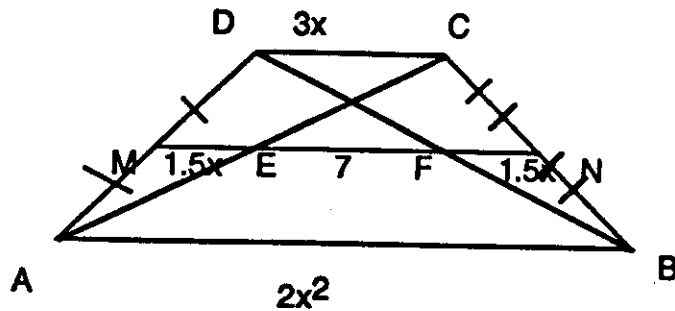
$$x = 4 \rightarrow 8(4) + 3y = 41 \rightarrow y = 3$$

$$8x + 3y = 41 \rightarrow y = -8/3x + 41 \rightarrow \perp : m = 3/8 \rightarrow y - 3 = 3/8(x - 4) \rightarrow y - 3 = 3/8x - 12/8$$

$$\rightarrow y = 3/8x + 12/8 \rightarrow 8y = 3x + 12 \rightarrow 3x - 8y = -12$$

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Team Question #6: 3.5



$$1.5x + 1.5x + 7 = \frac{1}{2}(2x^2 + 3x)$$

$$6x + 14 = 2x^2 + 3x \rightarrow 2x^2 - 3x - 14 = 0 \rightarrow (2x - 7)(x + 2) = 0 \rightarrow x = 3.5 \text{ or } x = -2 \text{ (reject)}$$

Team Question #7: $-3 + 3\sqrt{5}$

The equation for golden rectangles is: length / width = length + width / length

$$6/w = (6+w)/6 \rightarrow 36 = 6w + w^2$$

$$x^2 + 6x - 36 = 0 \rightarrow x = \frac{-6 \pm \sqrt{36 - 4(1)(-36)}}{2} \rightarrow x = -3 + 3\sqrt{5} \text{ or } x = -3 - 3\sqrt{5} \text{ (reject)}$$

Team Question #8: 12

Let $PQ = x$ and $QR = 21 - x$

$$h^2 + x^2 = 400 \text{ and } h^2 + (21 - x)^2 = 169 \rightarrow h^2 + 441 - 42x + x^2 = 169$$

$$h^2 + x^2 + 441 - 42x = 169$$

$$400 + 441 - 42x = 169$$

$$x = 16 \rightarrow h^2 + 16^2 = 400 \rightarrow h = 12$$

Team Question #9: 24 m

$$\text{volume of the cylinder} = 6^2(18)\pi = 648\pi$$

$$\text{volume of the cone} = \frac{1}{3}\pi(9^2)h = 27\pi h$$

$$27\pi h = 648\pi$$

$$h = 24$$

Team Question #10: 46

$$2(35 - 12) = 46$$

1. 165
2. 126
3. Isosceles and Right
4. 56
5. $3x - 8y = -12$
6. 3.5
7. $-3 + 3\sqrt{5}$
8. 12
9. 24
10. 46
11. (-2, 4)
12. $(4m - 3p)$ by $(m - 1)$
13. 65
14. $4\sqrt{41}$
15. $\sqrt{3}/1$