MARCH REGIONAL GEOMETRY TEAM QUESTIONS ANSWERS

3/11/95

1. $160\sqrt{2} + 50$

2. 25°

3. 12

4. $\frac{309}{5}$

5. 225

6. 10 m

7. 6

8. 20, 148

9. 40°

10. $\sqrt[3]{300}$ OR 6.7

11. $1152 + 1296\sqrt{3} + 576\pi$

12 450π

13. (200π - 400) square inches.

14. 360

15. 12

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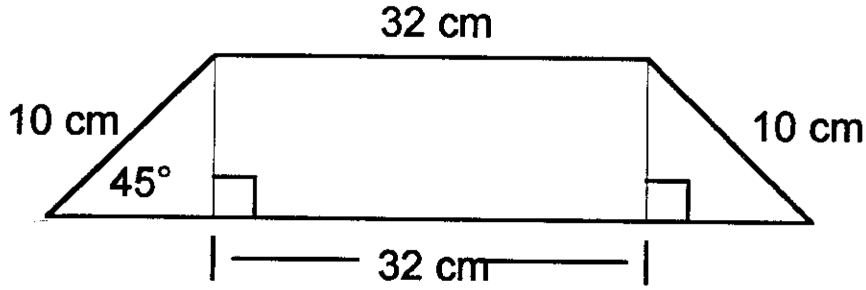
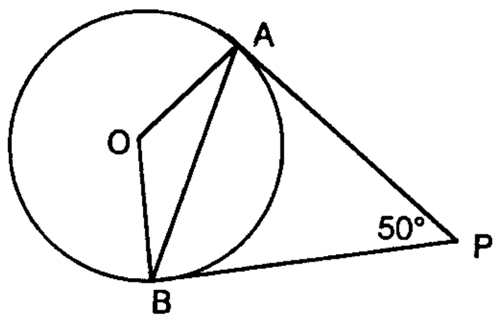
QUESTION # 1

An isosceles trapezoid has legs 10 cm long and one pair of base angles that are 45°. If the length of the shortest base is 32 cm, find the area of the trapezoid.



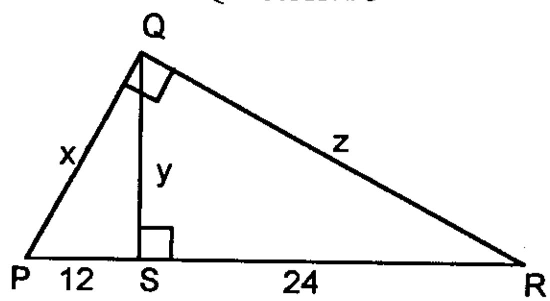
Using the 45-45-90 triangle relationship of the sides to the hypotenuse, we find the altitude to be $5\sqrt{2}$, and the two extra segments of the longer base to both be $5\sqrt{2}$ also. Using the formula, $A = \frac{1}{2}(b_1 + b_2)h$ with the lengths we have found, the area of the trapezoid is $160\sqrt{2} + 50$.

QUESTION # 2



Given circle O with tangent segments PA and PB. If angle APB is 50°, find the measure of angle OAB. The segments OA and OB are perpendicular to the tangent segments at the point of tangency. Since perpendicular lines form right angles and angle APB is 50°, and since the sum of the interior angles of a convex quadrilateral is 360°, angle AOB can be show to be 130°. Segments OA and OB are radii of the same circle and therefore congruent, making triangle BOA isosceles, and therefore angles OAB and OBA congruent or 25° each.

QUESTION # 3



Find $(x \bullet y) \div z$.

Using $\frac{12}{y} = \frac{y}{24}$, y is $12\sqrt{2}$. Now using either the geometric mean relationships or the Pythagorean relationship, we find $x = 12\sqrt{3}$, and $z = 12\sqrt{6}$ and therefore the solution is 12.

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QUESTION # 4

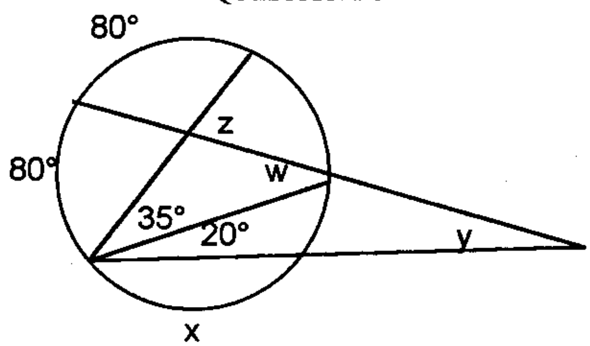
Consider a regular 12-gon. Let n represent the number of diagonals of the polygon. Let m represent the sum of the measures of the interior angles of the polygon. Let p represent the value of one exterior angle of the polygon.

Find $\frac{n + m}{p}$.

The sum of the interior angles is (n - 2)180, 18000. The value of one exterior angle is 360 divided by 12 or 40°.

The number of diagonals is $\frac{n(n - 3)}{2}$ or 54. Therefore the soulution is $\frac{309}{5}$.

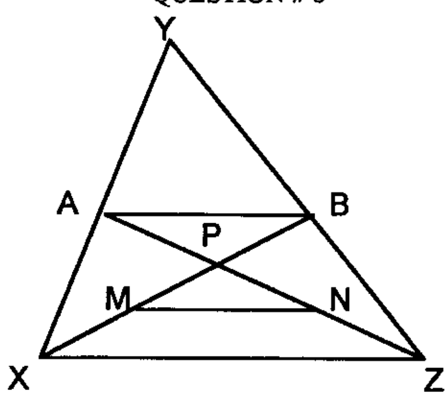
QUESTION # 5



Find w + x + y + z.

The value of w is one-half of 80 or 40. The value of the arc cut by the inscribed 35° angle is 70. The value of z is (70 + 80) ÷ 2 or 75. The value of the arc cut by the inscribed angle of 20° is 40. The value of y is (80 - 40) ÷ 2 or 20. The value of x is 360 - (80 + 80 + 70 + 40) or 90. The sum of the variables is 225.

QUESTION # 6



A and B are the midpoints of $\overline{XY}$ and $\overline{ZY}$ respectfully. M and N are the midpoints of $\overline{XP}$ and $\overline{ZP}$ respectfully. If AB = 10 m, then MN = ___?___ .

Since XB and ZA are medians they meet a point $\frac{2}{3}$ of the way from the vertex to the midpoint. Since M and N are midpoints of the $\frac{2}{3}$ segments MP = PB and NP=PA. The triangles can be proven congruent, and MN and AB are corresponding parts and therefore equal. MN = 10 m.

QUESTION # 7

The angles of a triangle are 5x, 3x + 21, and 7x + 9. The angles of a quadrilateral are 8y, 3y - 4, 6y + 3, and 5y + 9. Find $|x - y|$.

5x + 3x + 21 + 7x + 9 = 180, 15x + 30 = 180, 15x = 150, x = 10.

8y + 3y - 4 + 6y + 3 + 5y + 9 = 360, 22y + 8 = 360, 22y = 352, y = 16.

$|x - y| = |10 - 16| = |-6| = 6.$

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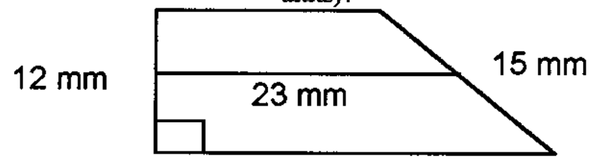
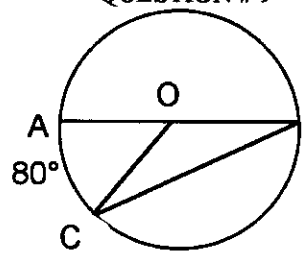
QUESTION # 8

A trapezoid with at least one right angle has legs of length 12 mm and 15 mm, and a median of 23 mm. If A is the area of the trapezoid and B is the perimeter of the trapezoid, what is the product of the values A and B (ignoring units)?



Since the median is the average of the lengths of the two bases, and since the trapezoid has a right angle, we can double the median and added it to the lengths of the legs to get the perimeter, we can also use the median in the formula for the area instead of the one-half the sum of the bases, to find the area. Additionally because the trapezoid has a right angle, the leg making the right angle with the bases is also the altitude of the trapezoid. Therefore the perimeter is 73 mm and the area is $276\ mm^2$ and the product is 20,148.

QUESTION # 9



Point O is the center. In the figure above, AOB is a diameter. Find the measure of angle OCB.

Angle AOC is 80° since a central angle has the same measure as its intersected arc, and consequently angle COB has a measure of 100° since it forms a linear pair. Angle OBC has a measure of 40° since it one-half of its intersected arc. Since the sum of the angles of a triangle is 180, subtracting we find angle OCB is 40°. Alternatively, since OB and OC are radii of the same circle the triangle COB is isosceles, and since angle OBC is 40°, so is angle OCB.

QUESTION # 10

A cone with a base of radius 10 and a height of 12 has the same volume as a sphere with a radius of ___?___.
Radical form or round to nearest tenth.

The cone has a volume of 400π. Using the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere, and setting these volumes equal, we find that $r = \sqrt[3]{300}$.

QUESTION # 11

Consider a circle with a radius of 24 cm. If A is the area of an inscribed square, B is the area of an inscribed equilateral triangle, C is the area of an inscribed hexagon, and D is the area of the circle, then find the sum of A + B + C + D.

The square has a diagonal of 48 cm. Using the formula that the area of a square is one-half the product of its diagonals we find the area to be $1152\ cm^2$. The equilateral triangle has a radius of 24 which means an altitude of 36 and using the 30-60-90 triangle relationship has a side length of $24\sqrt{3}$, and an area of $432\sqrt{3}\ cm^2$.. The hexagon is composed of 6 equilateral triangles each with a side of 24. Therefore the area of each equilateral triangle is $144\sqrt{3}\ cm^2$, and the area of the hexagon is $864\sqrt{3}\ cm^2$. The area of the circle is $576\pi\ cm^2$. And $A + B + C + D = 1152 + 1296\sqrt{3} + 576\pi$

QUESTION # 12

A circle circumscribed about a square has an area of $900\pi\ m^2$. Find the area of the circle inscribed within the same square. (UNITS ARE NOT NECESSARY.)

A circle inscribed in a square has one-half the area of the circle circumscribed about the square. therefore the area is $450\pi\ m^2$.

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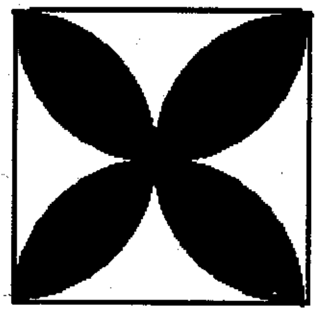
QUESTION # 13



The square above with sides of 20 inches has semicircles described on each of its sides. Find the area of the shaded region.

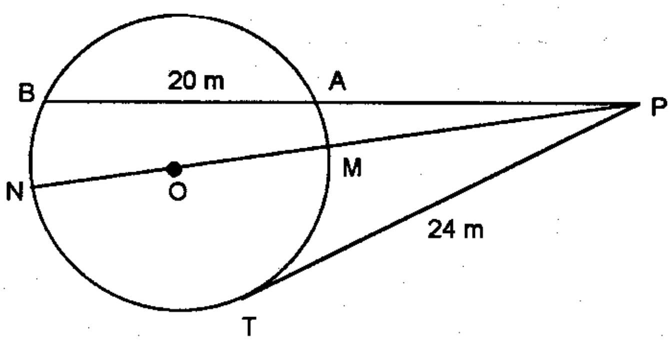
The figure is formed by eight segments of the circle of a 90° central angle, therefore the area of the shaded region is $8(25\pi - 50)$ square inches or
$(200\pi - 400)$ square inches.

QUESTION # 14

Consider a regular hexagon inscribed in a circle with a radius of 18 cm. If x is the length of the apothem of the hexagon, y is the radius of the hexagon, z is the measure of an interior angle of the hexagon, and w is the area of the hexagon, then find the value of $\frac{w \bullet z}{x \bullet y}$.

The apothem is the distance from the center of the hexagon to the midpoint of one of the sides, in this problem its length is $9\sqrt{3}$ cm. The radius of the hexagon is the same as the radius of the circle 18 cm. The measure of one interior angle of the hexagon is 120°. The area of the hexagon is $486\sqrt{3}$ cm^2. The value of the expression is 360.

QUESTION # 15



In the figure, circle O has a radius of 18 m. W is the length of PA, X is the length of PM, Y is the area of the circle, and Z is the circumference of the circle. Find the value of $\frac{W \bullet Y}{X \bullet Z}$.

Using the relationship $PA \bullet PB = PT^2$ we find PA = 16 m, and W = 16. Using the relationship $PM \bullet PN = PT^2$ we find that PM = 12 m, and X = 12. The area of the circle is 324π m^2. The circumference of the circle is 36π m. Therefore the value of the expression is 12.