Geometry — Team Solutions

March 24, 1993 — Plant

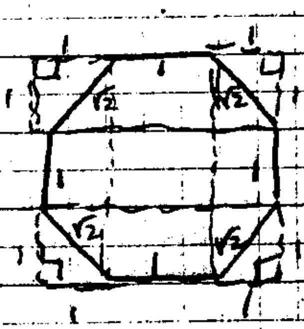
1. If $A(2,3)$, $B(4,7)$, $C(5,6)$, the slope of $AC = 1$, slope of $BC = -1$. Therefore, $\triangle ABC$ is a right $\triangle$.
$AC = \sqrt{18}$, $BC = \sqrt{2} \Rightarrow A = \frac{1}{2}(\sqrt{18})(\sqrt{2}) = \boxed{3}$.

2. The vol. of the hemisphere is $\frac{2\pi r^3}{3}$. The volume of the cone is $\frac{\pi r^2 h}{3}$. So, $\frac{16\pi}{3} - \frac{8\pi}{3} = \boxed{\frac{8\pi}{3}}$

3. The region bounded by the outer sphere is a circle of radius $= \sqrt{161}$, by the Pythag. Thm. So, area $= \boxed{161\pi}$.

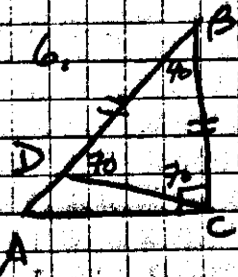
4. Every pair of lines determines a pt. of intersection. There are $\binom{6}{2}$ pairs of lines $= \boxed{15}$.

5.



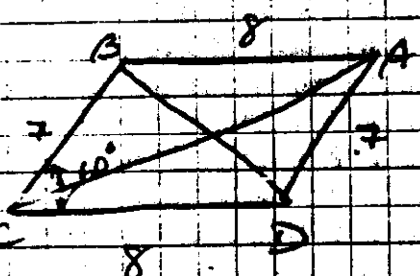
$A = 9$ squares $- 2$ squares or 5 squares $+ 2$ squares (All areas $= 1^2$)
So, $9 - 2 = \boxed{7}$.

6.



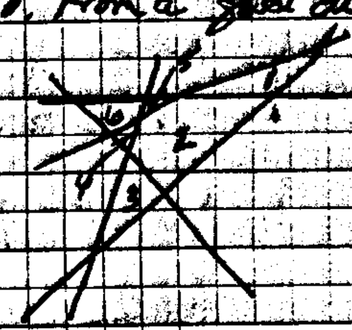
In isos $\triangle BDC$, $\angle BDC \cong \angle BCD = 70$
So $\angle ACD = \boxed{20^\circ}$

7.



$AC^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8 \cos 120^\circ$
$BD^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8 \cos 60^\circ$
The difference is $4 \cdot 7 \cdot 8 \cdot \cos 60^\circ$
$= \boxed{112}$.

8. From a good diagram



we get $\boxed{6}$.

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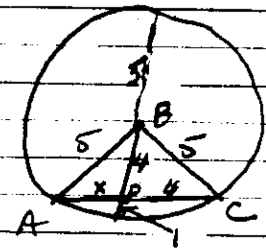
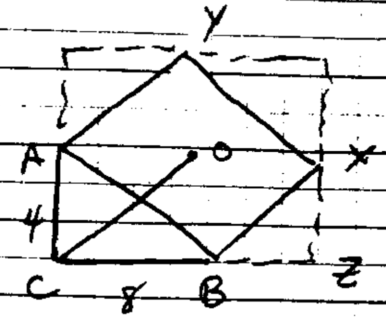
9. ~~Let B be the center of a circle w/ r=5.~~



$9 \cdot 1 = 4 \cdot x$

$\boxed{x = \frac{9}{4}}$

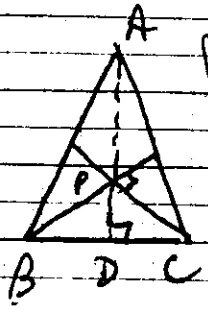
10.



~~Draw a $\triangle$s on ea. side of~~ the given sq. $\cong$ to $\triangle AB$

So, $CZ = 12$ and OC = $\frac{1}{2}$ its diagonal or $\boxed{6\sqrt{2}}$

11.



Since $BC = \sqrt{2}$, $PB = PC = 1$.

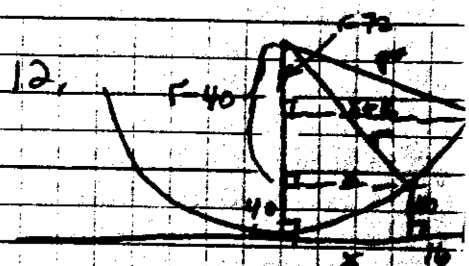
$PD^2 = PB^2 - BD^2 = 1^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

$\therefore PD = \frac{\sqrt{2}}{2}$

$AP = 2PD + AD = \frac{3\sqrt{2}}{2}$

So, Area $\triangle ABC = \frac{1}{2} \cdot \sqrt{2} \cdot \frac{3\sqrt{2}}{2} = \boxed{\frac{3}{2}}$

12,



$(r-72)^2 + (x+16)^2 = r^2$

$(r-40)^2 + x^2 = r^2$

Subt. & simp $\Rightarrow x = 2r - 120$

substituting back in, we get

$r^2 - 140r + 4000 = 0 \Rightarrow r = 100$

NOT TRUE

13. Draw CE, a median. A $\triangle$ CAE = $\frac{1}{2}$ A$\triangle$ACD

$\frac{BA}{CA} = \frac{3}{4}$; So, A$\triangle$BAE = A$\triangle$CAE = A$\triangle$ACD

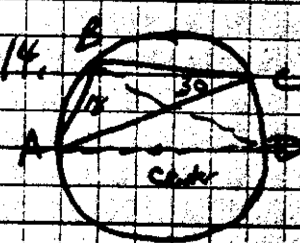
Omit

Area BEDC = A $\triangle$ACD

72 = A $\triangle$ACD $\Rightarrow$ A$\triangle$ ACD =

So, A $\triangle$BFE = $\frac{1}{2} \cdot$ = $\boxed{}$.

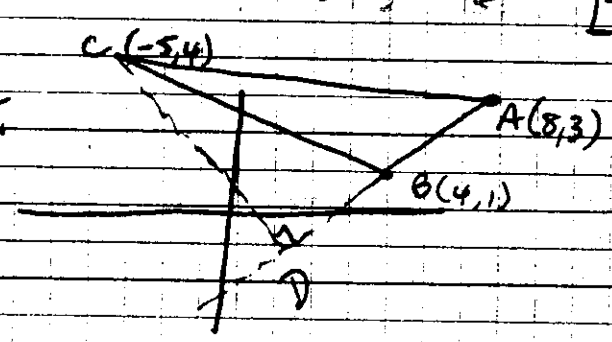
14.



$\angle BCA = \angle BDA = 30°$

$\triangle ABD$ is a rt $\triangle$, so AD = 2

15.



slope AB = $\frac{1}{2}$

eqn AB $\Rightarrow y - 3 = \frac{1}{2}(x - 8)$

slope CD = -2

eqn CD $\Rightarrow y - 4 = -2(x+5)$

solving: D = (-2, -2)

and CD = $\boxed{3\sqrt{5}}$

by the dist. formula.