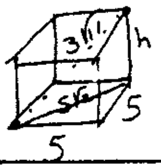
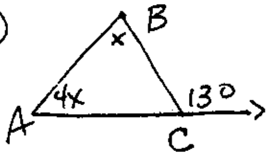
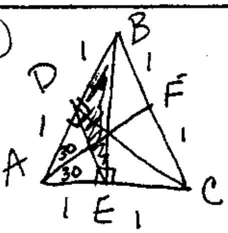
N.Plant ~~Geometry~~ Geometry Regional 1999 (March) Individual Solutions

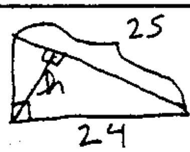
1) C — Sum of medians = $\frac{1}{2}$ perimeter = (28)

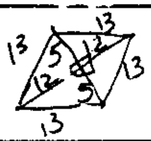
2) 
diag of base is $5\sqrt{2}$
$(3\sqrt{11})^2 = h^2 + (5\sqrt{2})^2$
(7) = h

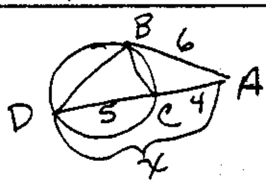
3) $77 = n\frac{(n-3)}{2}$; n = 14
$\frac{(14-2)(180)}{14}$ = ~~$1\frac{2}{7}$~~ $154\frac{2}{7}$

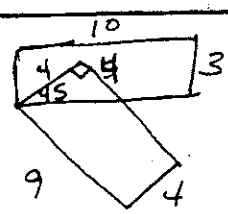
4) 
5x = 130
x = 26
m∠A = 4 · 26 = (104)

5) B 
a△ABE = $\frac{1}{2} \cdot \frac{s^2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$
△ADE is equilateral
a△ADE = $\frac{\sqrt{3}}{4}$
a△BDE = a△ABE − a△ADE = $\left(\frac{\sqrt{3}}{4}\right)$

6) 
$\frac{1}{2} \cdot 7 \cdot 24 = \frac{1}{2} \cdot 25 \cdot h$
$\frac{168}{25}$
(6.72) = h

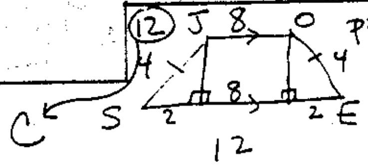
7) E 
$A = \frac{1}{2}d_1d_2$
$= \frac{1}{2} \cdot 10 \cdot 24$
= 120
120 = bh
120 = 13h
$\left(\frac{120}{13} = h\right)$

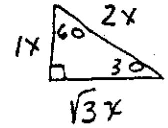
8) 
$6^2 = 4 \cdot x$
9 = x so DC = 5
$\frac{a\triangle ABC}{a\triangle BDC}$ = same altitudes so ratio of areas is ratio of bases
(4:5)

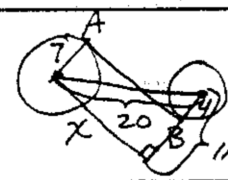
9) 
a 10×3 = 30
a 9×4 = 36
66 − 2a△
66 − 16 = (50)

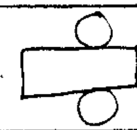
10) radius of ⊙ = 2, sides of hexagon = 2
p = 12

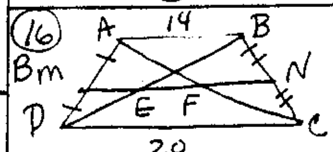
11) A
$288\pi = \frac{4}{3}\pi r^3$
6 = r;
$V = \frac{1}{3}\pi r^2 h$
$288\pi = \frac{1}{3}\pi \cdot 36h$
(24 = h)

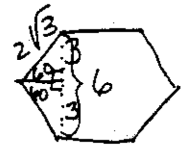
12) C 
p = 28, JS = OE = 4
$h = 2\sqrt{3}$
a⏢ = mh
$= 10 \cdot 2\sqrt{3}$
$(20\sqrt{3})$

13) D 
△ is 30, 60, 90
To find alt,
$\frac{1}{2}bh = \frac{1}{2}bh$
$\frac{1}{2} \cdot x \cdot \sqrt{3}x = \frac{1}{2} \cdot 2x \cdot h$
$\frac{\sqrt{3}}{2}x = h$; $\frac{\sqrt{3}x}{\frac{\sqrt{3}}{2}x} = \frac{2}{1}$

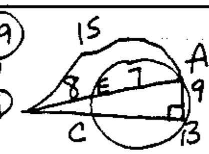
14) C 
Find AB.
Using Pythag,
$x^2 + 11^2 = 20^2$, $x = (3\sqrt{31})$

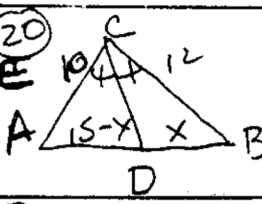
15) B 
To find V, need h.
r = 4
$24\pi = 2\pi rh$
3 = h
$V = \pi r^2 h$
$= 16\pi \cdot 3$
$= (48\pi)$

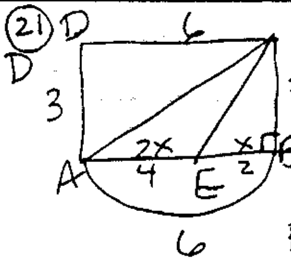
16) B 
mN = 17
$mE = \frac{1}{2} \cdot 14 = 7 = FN$
EF = 17 − 14 = 3
$\frac{EF}{mE} = \left(\frac{3}{7}\right)$

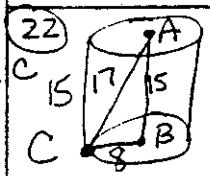
17) D 
$(p = 12\sqrt{3})$

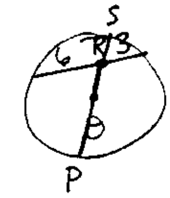
18) C side = x, diag = ~~x+6~~ and $x\sqrt{2}$
$x + 6 = x\sqrt{2}$, $(x = 6 + 6\sqrt{2})$

19) D 
DB = 12, Let DC = x.
8 · 15 = x · 12 ; x = 10, (BC = 2)

20) E 
$\frac{10}{15-x} = \frac{12}{x}$
$x = \frac{90}{11}$ = (8.2)

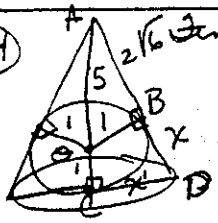
21) D 
3x = 6
x = 2, AE = 4, EB = 2
Using Pythag to find EC's
$(EC)^2 = 9 + 4 = \sqrt{13}$
$(AC)^2 = 9 + 36 = 3\sqrt{5}$
p△AEC = $(\sqrt{13} + 3\sqrt{5} + 4)$

22) C 
$TSA = 2\pi rh + 2\pi r^2$
$= 16\pi \cdot 15 + 2 \cdot 64\pi$
$= 240\pi + 128\pi$ = (368π)

23) C 
d = 11, $r = \frac{11}{2}$, Let RO = x, $PO = \frac{11}{2}$,
SR = $\frac{11}{2} - x$. Using 2 chord power theorem, $(6)(3) = \left(\frac{11}{2} + x\right)\left(\frac{11}{2} - x\right)$,
$18 = \frac{121}{4} - x^2$, $\left(x = \frac{7}{2}\right)$

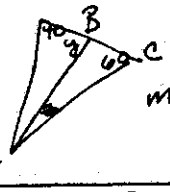
Geometry Regional 1999
2nd. Solutions continued.

(24) B



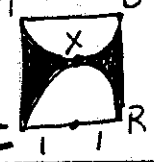
Find AB using Pythag.
$\triangle AOB \sim \triangle ABC$ by AA so
$\frac{1}{x} = \frac{5}{2\sqrt{6}+x}$, $x = \frac{\sqrt{6}}{2}$

(25) C



In $\triangle ABC$ $m\angle CBA = 180 - y$
$m\angle Z = 180 - (180 - y + 60)$
$= y - 60$

(26) B



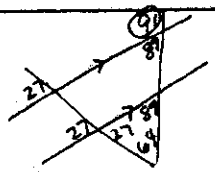
a shaded region $= a\square - a\odot$
$= 4 - \pi$

(27) B

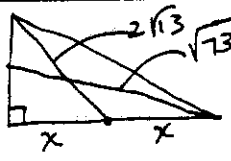
$x\sqrt{3} = \frac{s^2\sqrt{3}}{4}$
$s^2 = 4x$
$s = 2\sqrt{x}$,

$alt = 2\sqrt{x} \cdot \frac{1}{2}\sqrt{3} = \sqrt{3x}$

(28) C



(29) E



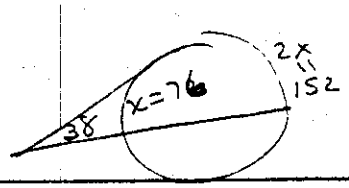
$\begin{cases} y^2 + (2x)^2 = 73 \\ (2y)^2 + x^2 = 52 \end{cases}$
$x = 4$, $y = 3$;
$8^2 + 6^2 = (hyp)^2$
$= 10$

(30) C

$x^2 + 2x + y^2 - 4y = 6$
$x^2 + 2x + \underline{1} + y^2 - 4y + \underline{4} = 6 + \underline{1} + \underline{4}$
$(x+1)^2 + (y-2)^2 = 11$, area $= 11\pi$

Team Solutions

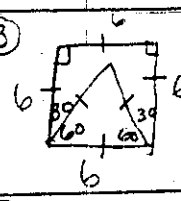
(1)



$2x - x = 76$
$x = 76$
3^{rd} arc $= 132$

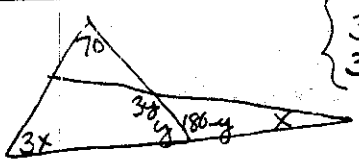
(2.) $A = 75$, $B = \frac{4}{3}\pi$ $(r = \frac{1}{3}\sqrt{3})$, $C = \frac{34}{360} \cdot 25\pi =$
$75 + \frac{4}{3}\pi + \frac{5}{2}\pi = 75 + \frac{23}{6}\pi = 87.0$

(3)



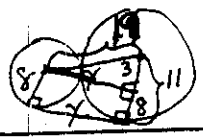
$a\square = 36$, $a\triangle = 9\sqrt{3}$
$36 - 9\sqrt{3} = 20.4$

(4)



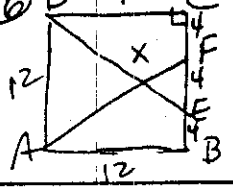
$\begin{cases} 3x + y = 110 \\ 3y = x + 180 - y \end{cases}$
$x - 4y = -180$
$3x + y = 110$
$m\angle A =$ (20, 50)

(5)



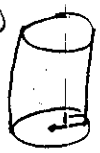
Using Pythag
$19^2 = 3^2 + x^2$
$4\sqrt{22}$

(6)



Find DE, use Pythag.
$DE = 4\sqrt{13}$. $\triangle ADX \sim \triangle EFX$ (4y!)
$\frac{DX}{EX} = \frac{3}{1}$ $\frac{4\sqrt{13} - x}{x} = \frac{3}{1}$, $x = \sqrt{13}$

(7)



$V = \frac{1}{3} \cdot 4\pi \cdot 6$
$= 8\pi$

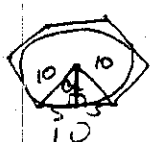
$V_{cyl} = \pi r^2 h$
$8\pi = \pi \cdot 1h$
$8 = h$

(8) $A = -\frac{3}{2} + \frac{4}{3} = -\frac{1}{6}$; $B = \frac{5}{2} + \frac{-7}{3} = \frac{1}{6}$
$A \div B = -1$

(9)

ratio of sides $\frac{9}{5}$, ratio of areas $\frac{9}{25}$
$\frac{9}{25} = \frac{x}{225}$; $x = 81$

(10) $9 = \frac{n(n-3)}{2}$, $n = 6$; $150\sqrt{3} = 6 \cdot \frac{s^2\sqrt{3}}{4}$
$100 = s^2$, $s = 10$



$a = 5\sqrt{3}$, $a\odot = (5\sqrt{3})^2\pi$
$= 75\pi$