FAMAT Regional Competition
Geometry Individual Test Solutions

March 1997

1. A
2. B
3. D
4. D
5. C
6. B
7. C
8. D
9. B
10. E
11. A
12. C
13. A
14. C
15. B
16. C
17. A
18. B
19. B
20. D
21. C
22. B
23. A
24. E
25. E
26. C
27. A
28. C
29. B
30. C

Plant

GEOMETRY INDIVIDUAL TEST SOLUTIONS

MARCH 1997

A 1. let $\angle A = 180 - x$

$\angle B = x$

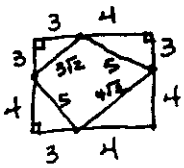
$\angle C = 90 - x$

$$\frac{180-x}{90-x} = \frac{7}{2}$$

$$630 - 7x = 360 - 2x$$

$$54^\circ = x$$

B 2.



$$3\sqrt{2} + 4\sqrt{2} + 5 + 5$$

$$7\sqrt{2} + 10$$

D 3. let $\angle APB = y = \angle BPC = \angle CPD$

let $\angle DPE = \angle EPF = x$

$3y + 2x = 180 \quad (x + y = 69) \cdot -2$

$-2y - 2x = -138$

$y = 42$

$42 + 42 + 42 + 2x = 180$

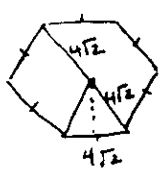
$x = 27$

$\angle APE = 3(42) + 27$

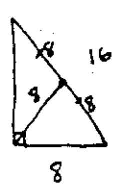
$= 153^\circ$

D 4. $\frac{270}{360} = \frac{3}{4} \cdot 2\pi \cdot 6 = 9\pi$

C 5.



B 6.



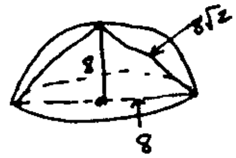
C 7. $360 - x - x = 58 \cdot 2$

$360 - 2x = 116$

$-2x = -244$

$x = 122$

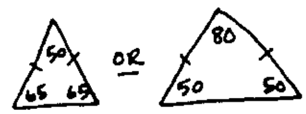
D 8.



LA of cone $= \pi r l = \pi(8)(8\sqrt{2}) = 64\pi\sqrt{2}$

LA of Hemisphere $= 2\pi(8)^2 = 128\pi$

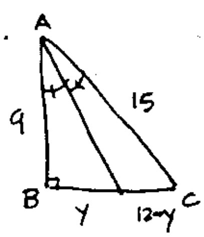
B 9.



65° OR 80°

E 10. could be a kite

A 11.



$$\frac{9}{15} = \frac{y}{12-y}$$

$5y = 36 - 3y$

$8y = 36$

$y = 9/2$

$9^2 + (9/2)^2 = AD^2$

$81 + \frac{81}{4} = AD^2$

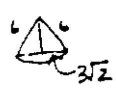
$\frac{324}{4} + \frac{81}{4} = AD^2$

$\frac{9\sqrt{5}}{2} = AD$

C 12.

creates 2 cones

$V = \frac{1}{3} B \cdot h$



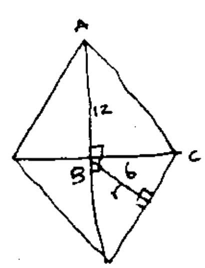
$V = \frac{1}{3}(3\sqrt{2})^2 \pi \cdot 3\sqrt{2}$

$V = \frac{1}{3}(9 \cdot 2)\pi \cdot 3\sqrt{2}$

$V = 18\sqrt{2}\pi \cdot 2$ cones

$V = 36\sqrt{2}\pi$

A 13.



$AC^2 = 6^2 + 144$

$AC = \sqrt{180}$

$6 \cdot 12 = 6\sqrt{5} \cdot r$

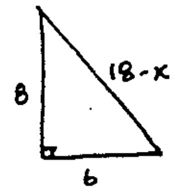
$\frac{6 \cdot 12}{6\sqrt{5}} = r$

$\frac{12}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = r$

$\frac{12\sqrt{5}}{5} = r$

A1

$\boxed{C}$ 14.



$x^2 + 6^2 = (18-x)^2$

$x^2 + 36 = 324 - 36x + x^2$

$-288 = -36x$

$8 = x$

So pole = 10

$\boxed{B}$ 15. $CF = 8$. $FP \perp AB$

$\triangle PFC$ is a rt $\triangle$ $FP = 2\sqrt{15}$

$SP = \sqrt{15}$

A of $\odot S = (\sqrt{15})^2\pi = 15\pi$

$\boxed{C}$ 16. $\angle DCF \cong \angle BCE$ by complements of same angle ($\angle FCB$) are $\cong$.

$\angle D \cong \angle CBE$, and $DC \cong BC$

so $\triangle FDC \cong \triangle EBC$, so $CE \cong CF$

so $\triangle CEF$ is isos.

$A = \frac{1}{2} CE^2 = 200$

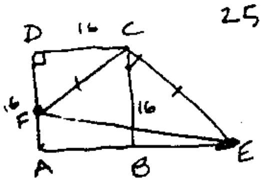
$CE^2 = 400$

$CB^2 + BE^2 = CE^2$

$256 + BE^2 = 400$

$BE^2 = 144$

$BE = 12$



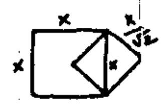
$\boxed{A}$ 17. The triangle formed is $\frac{1}{4}$ as large as the original. Shaded region equals a semi-circle $(r = \frac{s}{2})$ minus $2\triangle$s.

semi $\odot = \pi \left(\frac{s}{2}\right)^2 \cdot \frac{1}{2} = \frac{s^2\pi}{8}$

$\triangle = \frac{\left(\frac{s}{2}\right)^2\sqrt{3}}{4} \cdot 2 = \frac{s^2\sqrt{3}}{8}$

$\frac{\pi s^2}{8} - \frac{s^2\sqrt{3}}{8}$

$\boxed{B}$ 18.

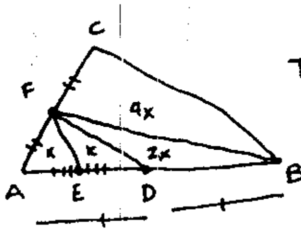


A of 1^{st} $\square = x^2$

A of 2^{nd} $\square = \frac{x}{\sqrt{2}} \cdot \frac{x}{\sqrt{2}} = \frac{x^2}{2}$

ratio $\frac{x^2}{\frac{x^2}{2}} = \frac{2}{1}$

$\boxed{B}$ 19.



The median of a $\triangle$ divides the area in $\frac{1}{2}$ so draw FD.

Let area of $\triangle AFE$ & $\triangle FED = x$

Let area of $\triangle FBD = 2x$ and area of $\triangle FCB = 4x$

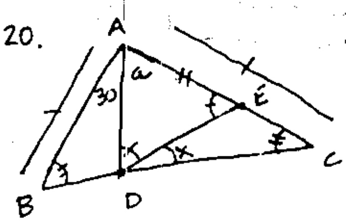
$3x = 63$

$x = 21$

$4 \cdot 21 = 84$

$84 \cdot 2 = 168$

$\boxed{D}$ 20.



$\angle BCA = \frac{1}{2}(180 - 30 - a)$

$75 - \frac{a}{2}$

$\angle DEA = \frac{1}{2}(180 - a)$

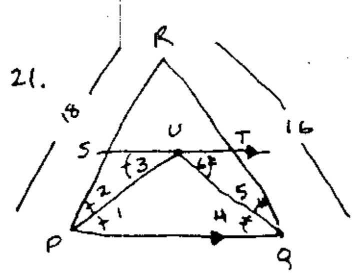
$90 - \frac{a}{2} = m\angle ADE$

in $\triangle ADC$ $a + (90 - \frac{a}{2}) + x + 75 - \frac{a}{2} = 180$

$a - a + 165 + x = 180$

$x = 15$

$\boxed{C}$ 21.



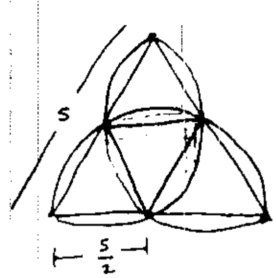
$\triangle RST \sim \triangle RPQ$

$\angle 1 \cong \angle 2$, $\angle 1 \cong \angle 3$, so $\angle 2 \cong \angle 3$

likewise $\angle 5 \cong \angle 6$

$\therefore PS \cong SU$ and $TQ \cong UT$

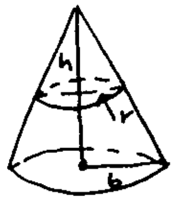
so $18 + 16 = 34$



$\boxed{B}$ 22.

$\pi\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}\pi$

$\boxed{A}$ 23.



Area of larger − Area of smaller = Area of smaller

$\frac{1}{3}\cdot 36\pi\cdot 8 - \frac{1}{3}\pi r^2\cdot h = \frac{1}{3}\pi r^2\cdot h$

$96\pi = \frac{2}{3}\pi r^2 h$

$144 = r^2 h \rightarrow \frac{3}{4}h = r$ by similar $\triangle$s

$144 = \left(\frac{3}{4}h\right)^2\cdot h$

$256 = h^3$

$4\sqrt[3]{4} = h$

$\boxed{E}$ 24. by Heron's formula

$s = \frac{8+9+15}{2} = 16$

$A = \sqrt{16(16-8)(16-9)(16-15)}$

$A = \sqrt{16(8)(7)(1)} = 8\sqrt{14}$

$\boxed{E}$ 25. $3x + 5x + 7x = 180$

$x = 12$

Largest $= 7\cdot 12 = 84$

$\boxed{C}$ 26. radius of circle (r)

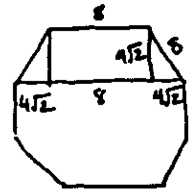
$r = \sqrt{(4-2)^2 + (3 - {}^-3)^2}$

$= \sqrt{4+36} = 2\sqrt{10}$

$c = 2\pi(2\sqrt{10}) = 4\pi\sqrt{10}$

$\boxed{A}$ 27. $81\pi - 36\pi = 45\pi$

$\boxed{C}$ 28.



Area of 2 Traps $= \frac{1}{2}(4\sqrt{2})(8 + 8 + 8\sqrt{2})$

$2\sqrt{2}\,(16 + 8\sqrt{2})$

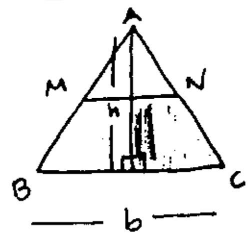
$2(32\sqrt{2} + 32)$

$64\sqrt{2} + 64$

Area of rect. $(8 + 8\sqrt{2})8 = 64\sqrt{2} + 64$

total $= 128\sqrt{2} + 128$

$\boxed{B}$ 29.



A of $\triangle ABC = \frac{1}{2}b\cdot h$

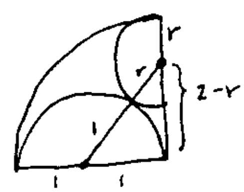
A sh. reg $= \frac{1}{2}\left(\frac{b}{2} + \frac{b}{4}\right)\frac{h}{2}$

$\left(\frac{b}{4} + \frac{b}{8}\right)\frac{h}{2}$

$\frac{3bh}{16}$

ratio of areas $\dfrac{\frac{3bh}{16}}{\frac{bh}{2}} = \frac{3}{8}$

$\boxed{C}$ 30.



$1^2 + (2-r)^2 = (1+r)^2$

$1 + (4 - 4r + r^2) = 1 + 2r + r^2$

$5 - 4r + r^2 = 1 + 2r$

$4 = 6r$

$\frac{2}{3} = r$