Armwood MARCH REGIONAL

# GEOMETRY INDIVIDUAL SOLUTIONS

3/11/95

| | | |
|---|---|---|
| 1. E | 11. E | 21. C |
| 2. D | 12. C | 22. D |
| 3. C | 13. C | 23. A |
| 4. D | 14. D | 24. C |
| 5. D | 15. C | 25. B |
| 6. C | 16. B | 26. B |
| 7. D | 17. D | 27. C |
| 8. C | 18. B | 28. C |
| 9. C | 19. C | 29. A |
| 10. C | 20. E | 30. A |

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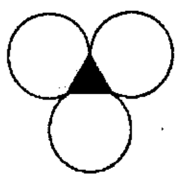
1. Not A because just because the three lines are parallel, does not mean that the distance between line one and two is the same as between two and three. Not B because there is no reason that tells you the lines and the transversal must be perpendicular. Not C because by definition the transversal must intersect the lines or it is not a transversal. Not D because then the lines would be perpendicular and B would then have to be true. Therefore since no correct answer is given the correct answer is E.



2. The chords connecting the points of tangency intercept arcs of 60 degrees, and therefore their lengths must be equal to the lengths of the radii of the circle. Applying the formula $A = \frac{s^2\sqrt{3}}{4}$ to the length of the sides which are each one-half of 20 or 10, we find the area of the triangle to be $25\sqrt{3}$ square meters of A.

3. Selection A is true because the triangles can all be proved using SSS from the theorem that the segment joining the midpoints of two sides of a triangle is parallel to the third and one-half its length. Choice B is true because both pairs of opposites sides are parallel, definition of a parallelogram (see theorem for A). Selection D is true because (see theorem for A) halves of unequals remain unequal in the same order. Choice C is definitely not true for any triangle, but especially in this case, for while PMKL is a trapezoid, the fact that the triangle is scalene makes the legs of the trapezoid, formed by taking half of two of the triangles sides, unequal, and therefore not an isosceles trapezoid.

4. The perpendicular bisectors of the sides of the triangle, choice D, meet at a point that is the center of the circle that would inscribe the triangle, since every point on each is equally distant from the endpoints of the side, the point of concurrence is the circumcenter and equally distant from the vertices.

5. Since the triangle is equilateral each side has a length of 24 cm. Applying the formula $A = \frac{s^2\sqrt{3}}{4}$, where s is the length of a side we find the area to be $144\sqrt{3}$ $cm^2$. Since this answer does not appear, the correct choice is D.

6. Since the sum of the measures of the central angles of any convex polygon must equal 360 degrees, and a regular polygon's central angles must all have the same measure, the value of each of those angles must divide 360 without a remainder. Choice C does not, therefore it cannot be the measure of a central angle of a regular polygon.

7. In a regular hexagon the length of a radius is the length of a side. The lateral surface area is the perimeter of the base times its height. Therefore since each side of the hexagon is 9 cm, and there are six sides the perimeter of the base is 54 cm. This perimeter, 54 cm, times the height or altitude, 8 cm, yields the lateral surface area of 432 $cm^2$, choice D.

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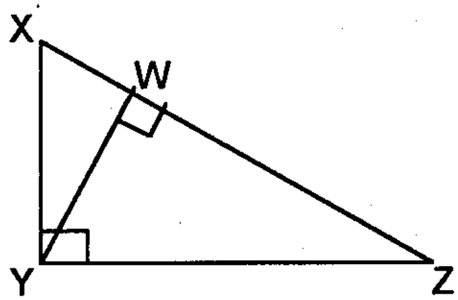
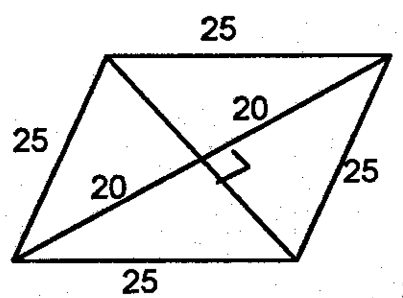
8. Since the diagonals of a square are congruent we may sue either diagonal. Applying the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, between either P and R or Q and S yields $13\sqrt{2}$, C.




9. When the altitude is drawn to the hypotenuse in a right triangle, the altitude and legs become geometric mean segments, That is $\overline{WY}$ is the geometric mean between $\overline{WX}$ and $\overline{WZ}$, $\overline{XY}$ is the geometric mean between $\overline{XW}$ and $\overline{XZ}$, and $\overline{YZ}$ is the geometric mean between $\overline{ZW}$ and $\overline{ZX}$. The only choice left $\overline{ZX}$, C, is not the geometric mean segment.

10. Using combinations, ${}_{33}C_2$ - 33 , yields 495 or choice C. Developed in geometry the formula: number of diagonals = $\dfrac{n(n-3)}{2}$ yields 495 choice C.

11. Given the perimeter you can always find the area of any regular polygon. Since none of the figures given is a regular polygon, the correct choice is E.




12. Since a rhombus is a parallelogram the diagonals bisect each other, and in a rhombus the diagonals are perpendicular to each other. It is therefore possible to find the measure of the other diagonal by first finding one-half using the Pythagorean Theorem ($a^2 + b^2 = c^2$). The length of one-half of the diagonal is 15 inches, and therefore the diagonal is 30 inches. Now using the formula, $A = \frac{1}{2} d_1 d_2$, area of a rhombus equals one-half diagonal one times diagonal two, we find the area of the rhombus to be 600 square inches, choice C.

13. Using the distance formula, as noted in #8, the distances between the points consecutively are all 10. This makes the figure a rhombus. The supporting theorem says that if both pair of opposite sides of a quadrilateral are congruent then the figure is a parallelogram, coupled with the definition of a rhombus determines the figure. Checking the lengths of the diagonals using the distance formula (see #8 above) determines that the diagonals are not congruent and therefore the figure is not a square. The best description in choice C.

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14. The altitude to the base of an isosceles triangle bisects the base. Since the base is 16 cm and the legs are congruent, each leg has a length of 12 cm. Using half the length of the base (8 cm) and the length of one leg along with the Pythagorean theorem, we can find the length of the altitude to be $4\sqrt{5}$ cm. Using the formula $A = \frac{1}{2}bh$ we find the area of the triangle to be $32\sqrt{5}$ square centimeters, choice D.

15. The third side of a triangle must have a length between the sum and difference of the lengths of the other two sides, this eliminates choices A and D. Using the Pythagorean relations we know that the third side would have a length of $4\sqrt{13}$ inches or a little over 14 inches if it were a right triangle. If the side is longer, then the angle opposite would have to be obtuse, so therefore the side must be less than it and choice C is correct.

16. Using the Pythagorean relationship we can determine that the length of the hypotenuse is 30 meters. Since a right triangle can be inscribed in a semicircle with the hypotenuse as a diameter, the median to the hypotenuse would be a radius of that circle, or one-half the hypotenuse, in this case 15 meters, choice B.

17. Since the lateral area of a cylinder is determined by the circumference times the height, and the circumference is $2\pi r$, its area is $2\pi rh$. The lateral area of a cone is determined by the circumference of the base times the slant height divided by 2, and since the circumference is $2\pi r$ is area is $\pi rl$. Setting these two area formula equal to each other, we are confronted with the height and slant height variables without a relationship also, therefore there is not enough information to determine the answer, choice D.

18. The two semicircles together make one circle with a diameter of 2 feet, and therefore a radius of 1 foot. The are of the shaded region is the area of the square $s^2$ minus the area of the circle $\pi r^2$ . or 4 feet minus $\pi$ or choice B.

19. Choice C, because only a rectangular parallelogram may be inscribed in a circle. Any regular polygon can be inscribed in a circle, therefore choice A is eliminated. Since the legs of an isosceles trapezoid are congruent, the perpendicular bisectors of those sides meet at a point that would be the center of the circumscribing circle, so choice B is eliminated. See #4 for why D is eliminated.

20. Choice E. Since the question asks if a diagonal, implying one diagonal, of a quadrilateral divides it into two congruent triangles, provides the possibility that the other does not. Since it may not, a possible figure might be a kite. Therefore there is insufficient information chose any of the selections.

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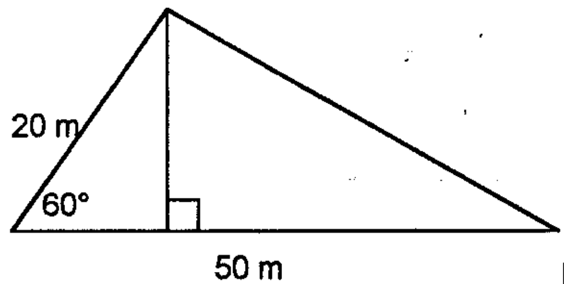
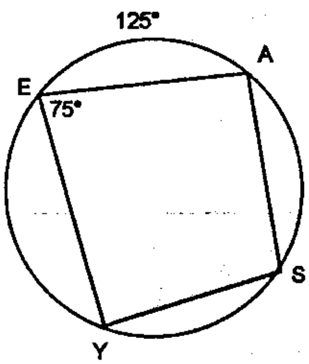



21. Dropping the altitude to the 50 m side produces a triangle that is 30-60-90 with the 20 m side the hypotenuse. Therefore the altitude to the 50 m side can readily be found to be $10\sqrt{3}$ m. Using this altitude to the 50 m base and the formula for the area of a triangle $\left(A = \frac{1}{2}bh\right)$ we find the area to be $250\sqrt{3}$ square meters, choice C.

22. Since a right triangle can be inscribed in a semicircle with the hypotenuse as a diameter, the median to the hypotenuse would be a radius of that circle, or one-half the hypotenuse, the two triangles would be isosceles, choice D.




23. Since the opposite angles of an inscribed quadrilateral are supplementary, angle S has a measure of 105°. Since an inscribed angle is one-half of its intercepted arc, the arc YEA has a measure of 210°. The arc YE plus the arc EA make up the are YEA., therefore, subtracting 125° from the 210° leaves 85° for the arc YE, choice A.

24. Choice C. Two intersecting planes determine a line.

25. The ratio of corresponding lengths of similar polygons squared is equal to the ratio of the areas of the similar polygons. Therefore: $\frac{24^2}{42^2} = \frac{80}{?}$. Solving for the unknown we find the area of the larger triangle to be 245 square inches, choice B.

26. If we call the angle A, then its supplement is 180 - A, and its complement is 90 - A. And we can then write the equation 180 - A = 4(90 - A) - 12. Solving for A we obtain the angle to be 56°, choice B.

27. $PT^2$ = PM x PN. Substituting in the know values we find PN to be 36 meters. Subtracting the 16 meters for PM, we then have MN = 20 meters, choice C.

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28. Since the radius of an equilateral triangle is two-thirds of the altitude, and the triangles formed are 30-60-90 triangles, the other length can readily be found and the area can be determined to be $36\sqrt{3}\ \text{cm}^2$.

29. Since the areas are equal we can set up the equation $s^2 = \frac{1}{2}bh$. Substituting in the know values, we find that the altitude (h) must be 60 inches, choice A.

30. The area of a sector of a circle is $\frac{n}{360}\pi r^2$, where $\pi r^2$ is the area of the circle. Substituting in the given values we find the area to be $7\ \text{cm}^2$, choice A.