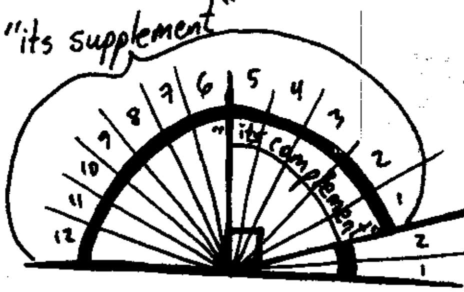
7

1. B



From the figure, 2:5 ratio of angle to its complement implies 2:12 = 1:6 ratio of angle to its supplement.

2. A The sum of the exterior angles of any polygon, one at each vertex is 360.

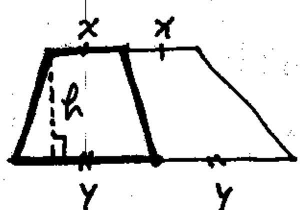
3. B $|x-3| \geq 8$, by definition of absolute value means:

$$x-3 \geq 8 \quad \text{or} \quad -(x-3) \geq 8$$
$$x \geq 11 \qquad\qquad -x+3 \geq 8$$
$$-x \geq 5$$
$$x \leq -5$$

We have:

"Two non-intersecting rays".

4. E Consider:



$$A_{BEFORE} = \frac{1}{2}h(2x+2y) = h(x+y)$$
$$A_{AFTER} = \frac{1}{2}h(x+y) = \frac{1}{2}A_{BEFORE}$$

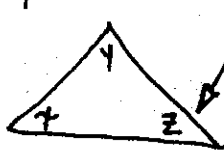
The area has been halved or "divided by 2".

5. A

$$\text{diameter} = \frac{\cancel{2}\sqrt{\pi}}{\pi} = \cancel{2}r, \quad \text{so radius} = \frac{\sqrt{\pi}}{\pi}$$

$$\text{AREA} \odot = \pi r^2 = \pi\left(\frac{\sqrt{\pi}}{\pi}\right)^2 = \pi\left(\frac{\pi}{\pi^2}\right) = \frac{\pi^2}{\pi^2} = 1$$

6. C If you recall that geometric mean $\leq$ arithmetic mean for any two numbers, then you know immediately that angle must be $\underline{60°}$. Otherwise, you can set up a system of equations:



$$z = \frac{x+y}{2} = \sqrt{xy}$$
$$z = 180-(x+y)$$

We have:

$$\frac{x+y}{2} = 180-(x+y)$$
$$(x+y) = 360-2(x+y)$$
$$3(x+y) = 360$$
$$(x+y) = 120$$

So $z = 180 - 120 = 60°$

If we insist on pounding out x & y, we get:

$$\sqrt{xy} = 60$$
$$xy = 3600$$

Since $x+y = 120$, then $y = 120-x$

Substituting, $x(120-x) = 3600$

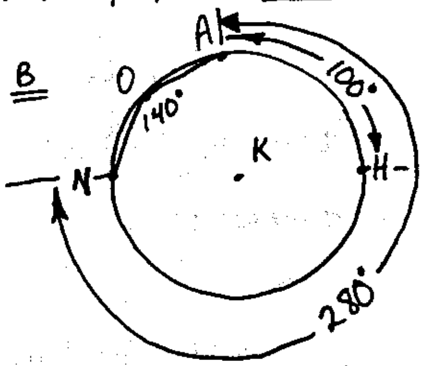
$$120x - x^2 = 3600$$

i.e., $x^2 - 120x + 3600 = 0$

$$(x-60)(x-60) = 0$$

and $x = 60°$ and $y = 60°$.

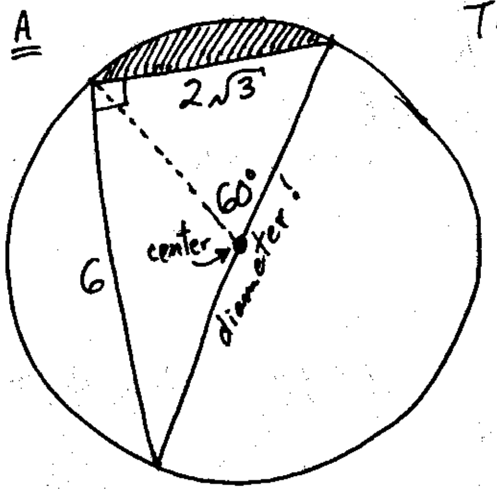
7. B



$280° - 100° = \underline{180°}$

It follows that $\widehat{NAH}$ is a SEMI-CIRCLE by definition of a semi-circle.

8. A



The shaded region has the smallest area.

$$\text{diameter}^2 = 6^2 + (2\sqrt{3})^2$$
$$= 36 + 4 \cdot 3$$
$$d^2 = 48$$
$$d = \sqrt{16 \cdot 3}$$
$$d = 4\sqrt{3}$$

so radius $= 2\sqrt{3}$

$A = A - A_{\triangledown}$

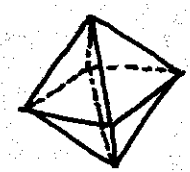
First, find $A_{\triangledown} = \frac{\text{side}^2\sqrt{3}}{4} = \frac{(2\sqrt{3})^2\sqrt{3}}{4} = \frac{\cancel{4} \cdot 3\sqrt{3}}{\cancel{4}}$

Next, find $A = \frac{n°}{360}(\pi r^2) = \frac{6\cancel{0}}{36\cancel{0}}\left(\pi\,(2\sqrt{3})^2\right)$

$= \frac{\pi}{6}(4 \cdot 3) = 2\pi$

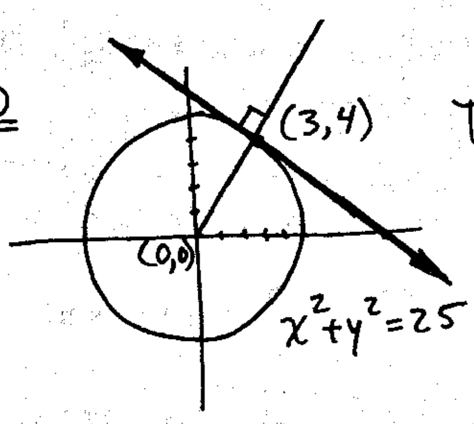
$= 2\pi - 3\sqrt{3}$

9. B An octahedron has 8 faces that are equilateral triangles:



Its surface area $= 8A = 8 \cdot \frac{1^2\sqrt{3}}{4} = 2\sqrt{3}$

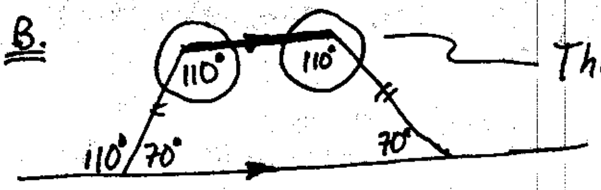
10. D



The slope of the radius to point (3,4) is $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{3-0} = \frac{4}{3}$.

(This is the slope of a line ⊥ to the tangent of the circle at (3,4) ☺!)

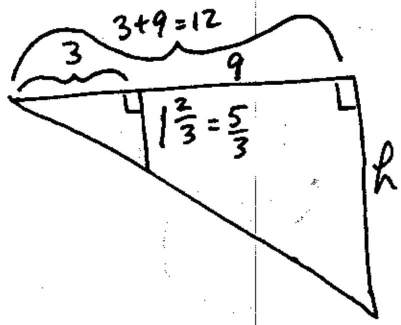
11. B.



The short base is included by interior angles measuring 110° each.

U

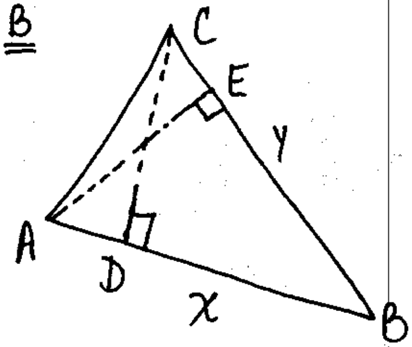
12. D



Use ~Δ's: $\frac{3}{12} = \frac{1}{4} = \frac{5/3}{h}$

$h = \frac{4\cdot 5}{3} = \frac{20}{3} = 6\frac{2}{3}$

13. B



$\frac{x}{y} = \frac{1}{2}$

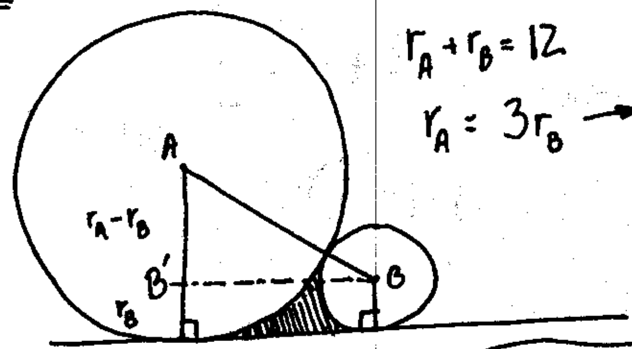
Use the area of the Δ:

$A_{\Delta ABC} = \cancel{\tfrac{1}{2}}(CD)x = \cancel{\tfrac{1}{2}}(AE)y$

We want to find $\frac{CD}{AE}$: $\frac{CD}{AE} = \frac{y}{x} = \frac{2}{1}$

14. D Not enough information to "fix" the measure of an interior angle. We need to know, for instance, that the polygon is regular.

15. C



$r_A + r_B = 12$

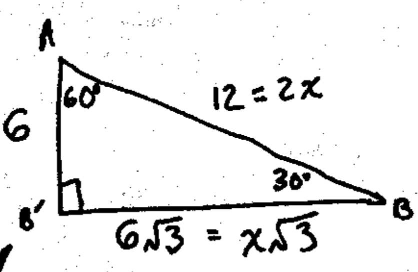
$r_A = 3r_B \rightarrow 3r_B + r_B = 12$

$4r_B = 12$

$r_B = 3$

$r_A = 3(3) = 9$

$x = 6$



(Notice $\Delta AB'B$ is a 30-60-90Δ.)

$A_{◢} = A_{⏢} - A_{◣} - A_{◔} \rightarrow A_{⏢} = A_{\square} + A_{\triangle}$

$= 3\cdot 6\sqrt{3} + \frac{1}{2}\cdot 6\cdot 6\sqrt{3}$

$= 36\sqrt{3}$

$A_{◣} = \frac{60}{360}(\pi 9^2) = \frac{\pi\cdot 3\cdot \cancel{3}\cdot 9}{2\cdot \cancel{3}} = \frac{27\pi}{2}$

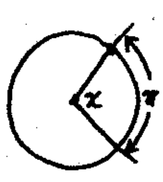
$A_{◔} = \frac{30+90}{360}(\pi 3^2) = \frac{\pi\cdot \cancel{3}\cdot 3}{\cancel{3}} = 3\pi$

$= 36\sqrt{3} - \frac{27\pi}{2} - 3\pi$

$= 36\sqrt{3} - \left(\frac{27\pi}{2} + \frac{6\pi}{2}\right)$

$= 36\sqrt{3} - \frac{33\pi}{2}$

16. B



$A_{\circ} = 40\cancel{\pi} = \cancel{\pi} r^2$

$\sqrt{40} = r$

$2\sqrt{10} = r$

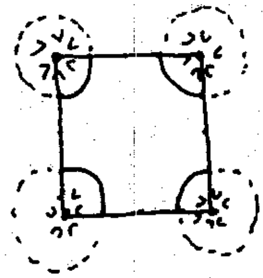
length of arc $= \frac{x}{360°}(2\pi r)$

$\cancel{\pi} = \frac{\cancel{2}\cancel{\pi}\, x\, (\cancel{2}\sqrt{10})}{\cancel{4}\cdot 90°}$

$\frac{x\sqrt{10}}{90} = 1$

$x = \frac{90}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{90\sqrt{10}}{10} = 9\sqrt{10}$

17. D Consider one square:



$\rightarrow A_{\square} = 4^2 = 16$

$A_{\text{✧}} = A_{\square} - A_{\odot}$

$= 16 - \pi$

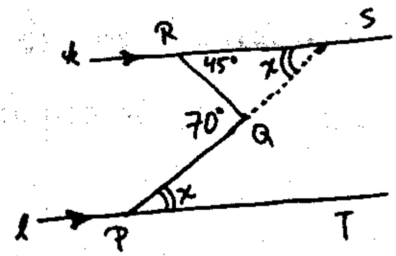
probability of not detonating a mine $= \dfrac{A_{\text{✧}}}{A_{\square}} = \dfrac{16-\pi}{16}$

$= 1 - \pi/16$

18. C Area Ratio $= \dfrac{4}{3} \Rightarrow$ height and edge ratio $= \dfrac{\sqrt{4}}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \Rightarrow$ volume ratio $= \dfrac{2^3}{(\sqrt{3})^3}$

so volume ratio $= \dfrac{8}{3\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{8\sqrt{3}}{9}$

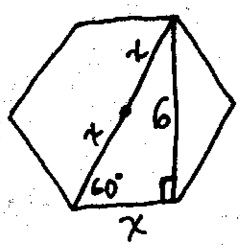
19. A



$70° = x + 45°$

$25° = x$

20. A

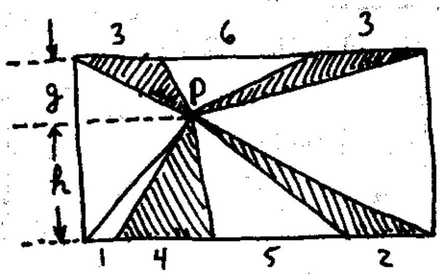


longer diagonal $= 2x$, but $x\sqrt{3} = 6$

$x = \dfrac{6}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$

So $2x = 2(2\sqrt{3}) = 4\sqrt{3}$

21. A



$\text{AREA}_{\square} = (g+h)12.$

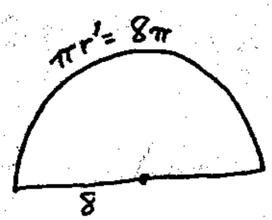
$\text{AREA}_{\times} = \frac{1}{2}g3 + \frac{1}{2}g3 + \frac{1}{2}h4 + \frac{1}{2}h2$

$= 3g + 3h$

$= (g+h)3$

FRACTION of area shaded $= \dfrac{A_{\times}}{A_{\square}} = \dfrac{\cancel{(g+h)}3}{\cancel{(g+h)}12} = \dfrac{1}{4}$

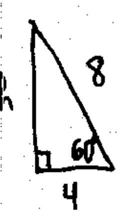
22. C



$\rightarrow$

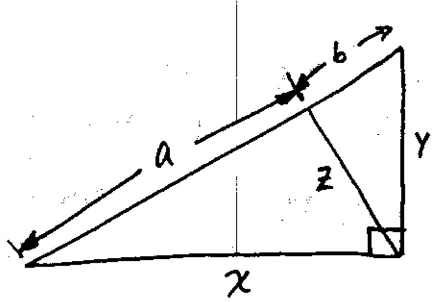
$8\cancel{\pi} = 2\cancel{\pi}r$

$4 = r$



$h = 4\sqrt{3}$ (30-60-90 Δ!)

23. C



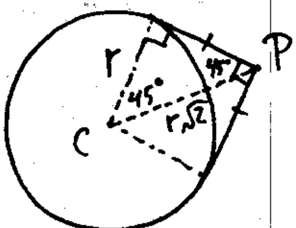
Using geometric means, we have:

$$\frac{z}{a} = \frac{b}{z} \qquad \frac{x}{a} = \frac{a+b}{x} \qquad \frac{y}{b} = \frac{a+b}{y}$$

$$z^2 = ab \qquad x^2 = a^2+ab \qquad y^2 = ab+b^2$$

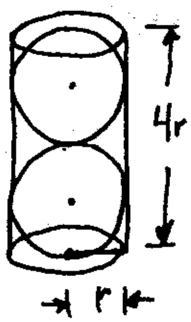
So $x^2+y^2+z^2 = a^2+3ab+b^2$

24. C



$$A_{\odot} = \cancel{\pi} r^2 = 2\cancel{\pi}$$
$$r = \sqrt{2}$$
$$PC = (\sqrt{2})\sqrt{2} = 2$$

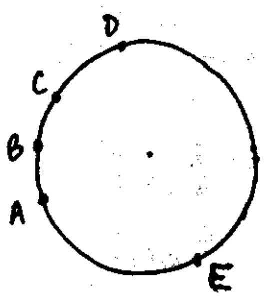
25. C



$$A_{\odot\odot} = 2 \cdot 4\cancel{\pi} r = 8\cancel{\pi}$$
$$\cancel{8} r = \cancel{8}$$
$$r = 1$$

$$V = \pi r^2 h = \pi r^2(4r) = 4\pi r^3 = 4\pi(1)^3 = 4\pi$$

26. D



5

$\overset{\frown}{AB}$	$\overset{\frown}{BC}$	$\overset{\frown}{CD}$	$\overset{\frown}{DE}$	$\overset{\frown}{EA}$
$\overset{\frown}{AC}$	$\overset{\frown}{BD}$	$\overset{\frown}{CDE}$	$\overset{\frown}{DEA}$	$\overset{\frown}{EB}$
$\overset{\frown}{AD}$	$\overset{\frown}{BDE}$	$\overset{\frown}{CDA}$	$\overset{\frown}{DEB}$	$\overset{\frown}{EC}$
$\overset{\frown}{ADE}$	$\overset{\frown}{BDA}$	$\overset{\frown}{CDB}$	$\overset{\frown}{DEC}$	$\overset{\frown}{EAD}$
$\cancel{\overset{\frown}{AE}}$	$\cancel{\overset{\frown}{BA}}$	$\cancel{\overset{\frown}{CB}}$	$\cancel{\overset{\frown}{DC}}$	$\cancel{\overset{\frown}{ED}}$

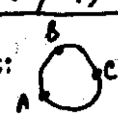
4

$4 \times 5 = 20$ arcs

If the "brute force" method is too distasteful to you, the "pattern method" works also:

1pt:  No Arc $< 360°$

2 pts:  2 ARCS

3 pts: 

$\overset{\frown}{AB}$, $\overset{\frown}{ABC}$, $\overset{\frown}{AC}$, $\overset{\frown}{BC}$, $\overset{\frown}{BCA}$, $\overset{\frown}{CAB}$ ⇒ 6 ARCS

4 pts: 

4

$\overset{\frown}{AB}$	$\overset{\frown}{BC}$	$\overset{\frown}{CD}$	$\overset{\frown}{DA}$
$\overset{\frown}{AC}$	$\overset{\frown}{BD}$	$\overset{\frown}{CDA}$	$\overset{\frown}{DAB}$
$\overset{\frown}{ABD}$	$\overset{\frown}{BCA}$	$\overset{\frown}{CDB}$	$\overset{\frown}{DAC}$
$\cancel{\overset{\frown}{AD}}$	$\cancel{\overset{\frown}{AB}}$	$\cancel{\overset{\frown}{BC}}$	$\cancel{\overset{\frown}{CD}}$

3

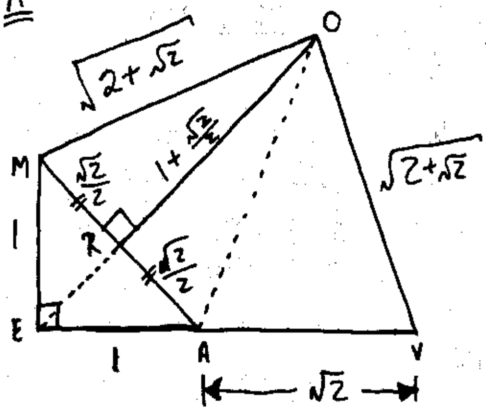
$3 \times 4 = 12$ arcs

The pattern:

# pts.	# arcs	difference between
1	0	
		2
2	2	
		4
3	6	
		6
4	12	
		8
5	20	

G.I. 3/14/92 SOLUTIONS

27. A



Steps:

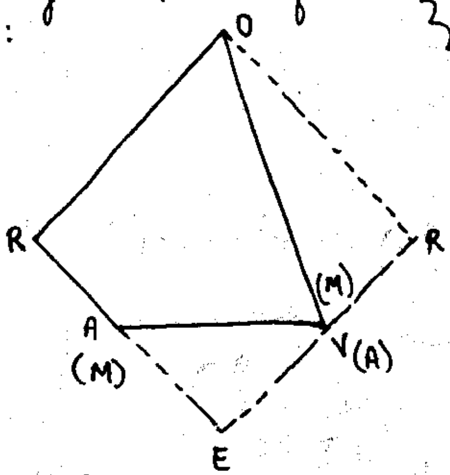
1. Notice $\triangle MEA$ is 45-45-90 and $MA = \sqrt{2}$
2. It follows that $MR = RA = \frac{\sqrt{2}}{2}$.
3. Then, by applying the converse of the Pyth. theorem to $\triangle MOR$, we discover that $\overline{OR} \perp \overline{MA}$ and that $\overline{OR}$ extended, will go through point E:

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(1+\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} + \left(1 + 2\cdot 1\frac{\sqrt{2}}{2} + \frac{2}{4}\right)$$
$$= 2+\sqrt{2}$$
$$= \left(\sqrt{2+\sqrt{2}}\right)^2$$

4. Notice $\triangle MOR \cong \triangle VOA$. Area of quadrilateral MOVE is: $A_{\triangle MEA} + 2A_{\triangle MOA}$

$$= \frac{1}{2}(1)(1) + \cancel{2}\left(\frac{1}{\cancel{2}}\right)(\sqrt{2})\left(1+\frac{\sqrt{2}}{2}\right)$$
$$= \frac{1}{2} + (\sqrt{2})\left(1+\frac{\sqrt{2}}{2}\right)$$
$$= \frac{1}{2} + \sqrt{2} + \frac{2}{2}$$
$$= \frac{3}{2} + \sqrt{2}$$

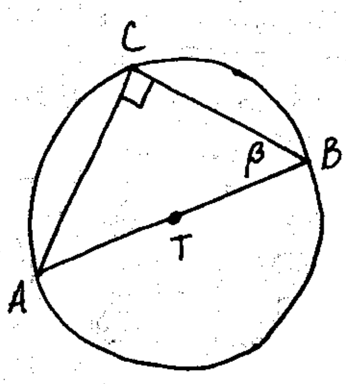
There is another way: Rearrange the parts of quad. MOVE:



The result is a square with side $OR = 1+\frac{\sqrt{2}}{2}$. Its area = area of MOVE $= \left(1+\frac{\sqrt{2}}{2}\right)^2 = 1 + 2\cdot 1\frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 = 1+\sqrt{2}+\frac{2}{4} = \boxed{\frac{3}{2}+\sqrt{2}}$

FOR FUN: The figure on the test paper is drawn in proportion. Cut out the pieces and verify that they form a square!

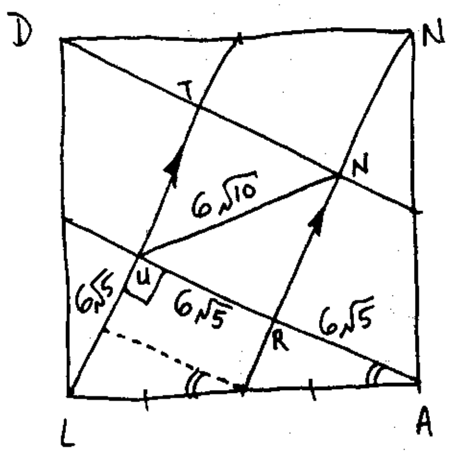
28. A

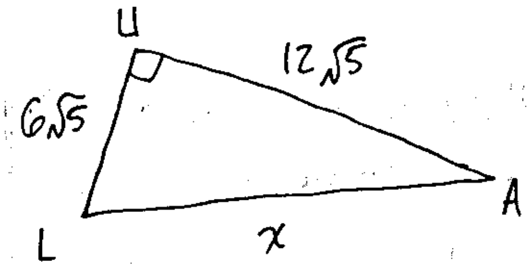


$$\frac{AC}{AB} = \frac{\text{opposite leg}}{\text{hypotenuse}}$$
$$= \sin\beta$$

G.I. 3/14/92 **SOLUTIONS**

29. B





Steps:

1. $(UR)\sqrt{2} = 6\sqrt{10} \Rightarrow UR = \frac{6\sqrt{10}}{\sqrt{2}} = 6\sqrt{5}$
2. Since $\overline{TU} \parallel \overline{RN}$ and they cut off equal ($\cong$) segments on $\overline{LA}$, it must be that $\overline{UR} \cong \overline{RA}$ and $UR = RA = 6\sqrt{5}$
3. By symmetry and since $TU = 6\sqrt{5}$, $UL = 6\sqrt{5}$
4. Apply the Pythagorean theorem to ΔLUA:

$$x^2 = (6\sqrt{5})^2 + (12\sqrt{5})^2$$
$$= 1\cdot 6^2\cdot 5 + 2^2\, 6^2\cdot 5$$
$$= 6^2\cdot 5(1+4) = 6^2\cdot 5^2 = (30)^2$$

$x = 30$ sweet...

30. B

Two ways to solve:

1. Brute force: plug in every (x,y) and see which one works.
2. Complete the square:

$$x^2 + y^2 - 6x + 14y + 22 = 0$$
$$\underbrace{x^2 + -6x + \boxed{(-3)^2}}_{x} + \underbrace{y^2 + 14y + \boxed{7^2}}_{y} = -22 + \underbrace{\boxed{(-3)^2}}_{x} + \underbrace{\boxed{7^2}}_{y}$$
$$(x-3)^2 + (y+7)^2 = -22 + 9 + 49$$

That is, the center is at $(3,-7)$ and the radius is:

$$\begin{array}{r} 58 \\ -22 \\ \hline 36 \end{array} \qquad \sqrt{36} = 6$$

9