Team Solutions!

(1)
$y = x(210-2x) = 210x - 2x^2$
$y' = 210 - 4x = 0$
$x = 52.5$
x, $210-2x$
$y(52.5) = \boxed{5512.5}$

(2) $\int_0^6 3x^3 - kx^2 + \frac{k}{2}x + k$
$= \frac{3}{4}x^4 - \frac{k}{3}x^3 + \frac{kx^2}{4} + kx \Big|_0^6$
$= \left[\frac{3}{4}(1296) - \frac{k}{3}(216) + \frac{36k}{4} + 6k\right] - 0$
$= 972 - 72k + 9k + 6k = 744$
$\Rightarrow -57k = -288$
$\Rightarrow k = \boxed{4}$

(3) A. $\frac{1}{6}\int_0^6 (x^3 - 2x^2 + 3x + 5)\,dx$
$= \frac{1}{6}\left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{3x^2}{2} + 5x \Big|_0^6\right]$
$= \frac{1}{6}(264) = 44$
B. $y' = 3x^2 - 4x + 3 \,\Big|_{x=0} = 3$
C. $\frac{f(7) - f(2)}{5} = \frac{271 - 11}{5} = 52$
$3x^2 - 4x + 3 = 52 \Rightarrow x \approx 5$
D. $y'' = 6x - 4 = 0 \Rightarrow x = 2/3$
$ABCD = \boxed{440}$

(4) $\theta = \frac{5\pi}{6} - -\frac{2\pi}{3} = \frac{3\pi}{2}$
$r^2 = 3^2 + 4^2 - 2(3)(4)\cos\frac{3\pi}{2}$
$r^2 = 25$
$\text{Area} = \pi r^2$
$= \boxed{25\pi}$

(5) $(x-5)^2 + (y-6)^2 = 9$
$x^2 + y^2 - 10x - 12y + 52 = 0$
$y = \pm\sqrt{-x^2 + 10x - 16} + 6$
$\frac{dy}{dx} = \frac{\mp(x-5)}{\sqrt{-x^2+10x-16}}$

(5 (con't)) $\frac{dy}{dx}(5) = 0$, $\frac{dy}{dx}(6) = \frac{\sqrt{2}}{4}$
$y - (6 - 2\sqrt{2}) = \frac{\sqrt{2}}{4}(x-6)$
$y = \frac{\sqrt{2}}{4}(x-6) + 6 - 2\sqrt{2} = 9$
$\Rightarrow x = 22.485,\ y = 9$
$\boxed{(22.485, 9.000)}$

(6) $f''(x) = x^2 + 3$
$f'(x) = \frac{x^3}{3} + 3x + C$, $f'(6) = 100$
$\frac{6^3}{3} + 3(6) + C = 100 \Rightarrow C = 10$
$f'(x) = \frac{x^3}{3} + 3x + 10$
$f(x) = \frac{x^4}{12} + \frac{3x^2}{2} + 10x + C$, $f(3) = 57$
$\frac{3^4}{12} + \frac{3(3)^2}{2} + 10(3) + C = 57$
$\Rightarrow C = 3/4$
$f(x) = \frac{x^4}{12} + \frac{3x^2}{2} + 10x + 3/4$
$f(5) = \boxed{140.333}$

(7) A - true, B - true, C - false: while the statement is true for some value, it is not necessarily true for $f'(c)$.
D - true
E - false, $f(x)$ may be piecewise if conditions A+B are met
F. - false

(8) A. $V = 2\int_0^{1.272} \frac{1}{2}\left(\frac{1+x^2-x^4}{2}\right)^2 \pi\,dx = 1.105$
B. $V = 2\int_0^{1.272} (1+x^2-x^4)^2\left(\frac{\sqrt{3}}{4}\right)dx = 1.219$
C. $V = 2\int_0^{1.272} (1+x^2-x^4)^2\,dx = 2.815$
$A+B+C = \boxed{5.139}$

(9) A. $P(0) = \frac{5000}{1+e^0} = 2500$, B. $P(\infty) = \frac{5000}{1+e^{-\infty}} = 5000$
C. $3500 = \frac{5000}{1+e^{-0.285t}} \Rightarrow t = 2.97$
$A+B+C = \boxed{7502.97}$

(10)
$d = \sqrt{(x-5)^2 + (x^2)^2}$
$= \sqrt{x^4 + x^2 - 10x + 25}$
$\frac{dd}{dx} = \frac{2x^3 + x - 5}{\sqrt{x^4 + x^2 - 10x + 25}} = 0$
$\Rightarrow x = 1.235$
$\Rightarrow d = \boxed{4.062}$

Calculus
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Solutions

(11) $f(6) = 0+1 = 1$, $h(1) = 2 - 3 = -1$,
$g(-1) = -1 - 5 = -6$, $f(-6) = \underline{6}$
$g(1) = 1 + 3 = 4$, $h(4) = 2 - 3 = -1$,
$h(-1) = -2 + 2 = 0$, $f(0) = -6$,
$h(-6) = -2 - 1 = -3$, $g(-3) = \underline{-3}$
$6 + (-3) = \boxed{3}$

(12) $d^2 = l_1^2 + l_2^2 - 2(l_1)(l_2)\cos\theta$
$\cancel{2}d\frac{dd}{dt} = \cancel{2}(l_1)(l_2)\sin\theta\frac{d\theta}{dt}$
$\frac{dd}{dt} = \frac{(l_1)(l_2)}{d}\sin\theta\frac{d\theta}{dt}$
$\theta = \pi/3 \Rightarrow d = \sqrt{7}$
$\frac{dd}{dt} = \frac{(2)(3)}{\sqrt{7}}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{7\pi}{12}\right)$
$\boxed{\frac{dd}{dt} = \frac{\pi\sqrt{21}}{4}}$

(13) $V = \int_4^9 2\pi r h = 2\pi\int_4^9 x\sqrt{x}\,dx$
$= 2\pi\left[\frac{2x^{5/2}}{5}\Big|_4^9\right] = \frac{422}{5}(2\pi)$
$= \boxed{\frac{844\pi}{5}}$

(14) $L = \int_0^3 \sqrt{9x^4 - 30x^2 + 26}\,d$
$L = 21.087$
$\frac{21.087 \text{ miles}}{20 \text{ min}} = 63.261 \text{ mph}$
$\approx \boxed{63 \text{ mph}}$

(15) $x^3 = \sqrt{x} \Rightarrow x = 0$ or $x = 1$
$A = \int_0^1 \sqrt{x} - x^3$
$= \frac{2}{3}x^{3/2} - \frac{x^4}{4}\Big|_0^1$
$= \frac{5}{12} - 0$
$= \boxed{5/12}$