Question 1 Calculus Team Round March Regional

$$A = \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$$

$$B = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \cos(k\pi)$$

Find AB

Question 2 Calculus Team Round March Regional

Use a linearization of $y = \sqrt{x}$ to approximate $\sqrt{88}$ using the fact that $\sqrt{81} = 9$.

Question 3 Calculus Team Round March Regional

$$A = \lim_{x \to 0} \frac{\sin(2x) + \cos x - 1}{x}$$

$$B = \lim_{x \to \infty} \sqrt{x^2 - x} - x$$

$$C = \lim_{x \to 0} x^{\ln \frac{1}{x}}$$

$$D = \lim_{x \to \infty} \frac{x^2 - 3x}{3x^2 + 2}$$

Find $AB + C + \frac{1}{D}$

Question 4 Calculus Team Round March Regional

A bridge is formed by a parabolic arch. At the top it is 12' high, and at the ground, it is 48' wide. How tall is it at a point 12' from one of the bases at the end?

Question 5 Calculus Team Round March Regional

If $f(x) = ax^3 + bx^2 + cx + d$ has a local maximum at $(1, 1)$ and a local minimum at $(-1, -2)$, find $a + b + c + d$.

Question 6 Calculus Team Round March Regional

Let $f(x) = x^3 + 3x^2 - 6x + 11$ and $g(x) = \ln|x|$.

$$A = f(g'(-1))$$

$$B = g(f'(-1))$$

$$C = \int_{-2}^{1} f'(x)g'(f(x))\,dx$$

$$D = \frac{d}{dt}\int_{1}^{t^2} g(x)\,dx \qquad \text{evaluated at } t = 3$$

Find $A(B + C) - D$

Question 7 Calculus Team Round March Regional

$A = \frac{dy}{dx}$ of $x^2 + y^2 = 4$ at $(\sqrt{2}, \sqrt{2})$

$B = \frac{dy}{dx}$ of $\sin y + sinx = \sin x \sin y$ at (π, π)

$C = \frac{dy}{dx}$ of $2e^{xy} + e^x e^y - e^x - e^y = e^{xy+1}$ at $(1, 1)$

Find $A + B + C$

Question 8 Calculus Team Round March Regional

Over what interval(s) is $f(x)$ increasing and concave down if $f(x) = 4x^2 - x^4$?

Question 9 Calculus Team Round March Regional

What is the maximum volume of a right circular cylinder that is inscribed in a sphere with radius 1?

Question 10 Calculus Team Round March Regional

Evaluate

$$\int_0^1 \int_0^y x^2 y^2 \, dx \, dy$$

Question 11 Calculus Team Round March Regional

If $\lim_{x \to \infty} \sqrt{Ax^2 + Bx} - Cx = 2$, find $\frac{BC}{A}$

Question 12 Calculus Team Round March Regional

Ray is paddling in a boat at 6mph. He is one mile off shore (perpendicular distance), and two miles downstream from his home. Ray will paddle at an angle until he reaches shore, and will then jog the rest of the way home. If Ray jogs at 10mph, what is the minimum time in which he can get home (in minutes)?

Question 13 Calculus Team Round March Regional

$$A = \int \frac{dx}{x^2+6x+25}$$

$$B = \int \frac{dx}{x^2-6x-27}$$

Find $12(A + B)$

Question 14 Calculus Team Round March Regional

What is the coefficient of the $x^{-5}y^3$ term of the expansion of $(x+y)^{-2}$?

Question 15 Calculus Team Round March Regional

Evaluate

$$\int_0^\infty e^{-x^2}\,dx$$