30. A - The equation can be set up that $T = T_0 + Ce^{kt}$, where $T_0 = 20$, the ambient temperature. A suitable set of values for C and k is $C = 32$ and $k = \frac{1}{10}\ln\frac{3}{4}$, when t is measured in minutes. Thus, at $t = 30$, $T = 20 + 32(\frac{3}{4})^3 = 20 + \frac{27}{2} = 33.5$, A.

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1. 2 - $A = \frac{1}{1-\frac{2}{3}} = 3$ and $B = \frac{1}{1-(-\frac{1}{2})} = \frac{2}{3}$, so $AB = 2$.

2. $\frac{169}{18}$ - $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$. If $x = 81$ and $\Delta x = 7$ then $f(88) \approx \sqrt{81} + (\frac{1}{2\sqrt{81}})(7) = 9 + \frac{7}{18} = \frac{169}{18}$.

3. 2 - By L'Hôpital's Rule, the limit of A is the same as $\lim_{x\to 0} 2\cos(2x) - \sin x = 2$. For B, multiplying the expression by $\frac{\sqrt{x^2-x}+x}{\sqrt{x^2-x}+x}$ yields $\frac{-x}{\sqrt{x^2-x}+x}$. The limit as x tends to ∞ is $-\frac{1}{2}$ so $AB = -1$. In C, an infinitesimal quantity, x, is beging raised to an infinite quantity, $\ln\frac{1}{x}$, so the limit is just zero. Finally, D tends to $\frac{1}{3}$. Therefore, $AB + C + \frac{1}{D} = -1 + 0 + 3 = 2$.

4. 9 - The focus of the arch is on the ground directly below the center. This is apparent because the width on the ground is exactly four times the hight, so the base of the arch forms the latus rectum. Therefore, the equation of the arch is $48(y-12) = -x^2$. Thus, when $x = 12$ or $x = -12$ (at 12' from one of the bases), $y = 12 - \frac{144}{48} = 9$.

5. 1 - The local extrema occur at $x = \pm 1$, so $f'(\pm 1) = 0$. $f'(x) = 3Ax^2 + 2Bx + C$. Therefore, $B = 0$ and $C = -3A$. Now, $f(1) = 1$, so $A - 3A + D = 1$, thus $D - 2A = 1$. In addition, $f(-1) = -2$ so $-A + 3A + D = -2$, thus $D + 2A = -2$. Solving the linear system, we conclude that $D = -\frac{1}{2}$ and $A = -\frac{3}{4}$, so $C = \frac{9}{4}$. Then, $A+B+C+D = \frac{6}{4} - \frac{1}{2} = 1$.

6. $7\ln 3$ - For A, $g'(-1) = -1$ and $f(-1) = 19$. For B, $f'(-1) = -9$ and $g(-9) = \ln 9$. For C, it must be noticed that this is the chain rule, and the integral will be $g(f(x))$ from -2 to 1, which is $\ln 9 - \ln 27 = \ln\frac{1}{3}$. Therefore, $B + C = \ln 3$. Finally, by the chain rule, $D = 2tg(t^2) = 6\ln 9$ when $t = 3$, which can also be written as $12\ln 3$. $A(B+C) = 19\ln 3$, so $A(B+C) - D = 7\ln 3$.

7. -3 - Because the relations in A, B, and C are all symmetric with respect to $y = x$ and are continuous and differentiable, the curves must be normal to $y = x$ at all intersecion points. Therefore, the slope of the tangent line to each curve at the points given (all of which are on $y = x$) must be -1 for all relations. Therefore, $A+B+C = -3$. Alternately, the students may have worked out all of the implicit derivatives by hand and plugged the points in to reach the same conclusion of $A + B + C = -3$.

8. $(-\infty, -2\sqrt{2}) \cup (\frac{\sqrt{6}}{3}, 2\sqrt{2})$ - $f(x)$ is increasing when $f'(x) > 0$, which occurs on $(-\infty, -2\sqrt{2})$ and $(0, 2\sqrt{2})$. $f(x)$ is concave down when $f''(x) < 0$, which occurs on $(-\infty, -\frac{\sqrt{6}}{3})$ and $(\frac{\sqrt{6}}{3}, \infty)$. The intersection of these intervals yields $(-\infty, -2\sqrt{2}) \cup (\frac{\sqrt{6}}{3}, 2\sqrt{2})$.

9. $\frac{4\pi\sqrt{3}}{9}$ - Let the radius of the base of the cylinder be given by r and the height by h. Therefore, the volume is given by $V = \pi r^2 h$. Because the cylinder is inscribed in a unit sphere, we can find a relation between r and h, that $r^2 + (\frac{h}{2})^2 = 1$. Substituting $r^2 = 1 - \frac{h^2}{4}$ into V yields $V = \pi h - \pi\frac{h^3}{4}$, which is a function of the height. Now we can differentiate V with respect to h to find the relative extrema. $V'(h) = \pi - \frac{3}{4}\pi h^2$. The height must be positive, so the solution to $V'(h) = 0$ is found to be $h^2 = \frac{4}{3}$, so $h = \frac{2\sqrt{3}}{3}$. Thus, $V = \frac{2\pi\sqrt{3}}{3} - \frac{2\pi\sqrt{3}}{9} = \frac{4\pi\sqrt{3}}{9}$.

10. $\frac{1}{18}$ - This a rather simple iterated integral. First the inside integral must be done, which leaves $\int_0^1 (\frac{1}{3}x^3y^2]_{x=0}^{y}\,dy = \int_0^1 \frac{1}{3}y^5\,dy$. Then it can be integrated normally to yield $(\frac{1}{18}y^6]_{y=0}^{1} = \frac{1}{18}$.

11. 4 - Multiplying by $\frac{\sqrt{Ax^2+Bx}+Cx}{\sqrt{Ax^2+Bx}+Cx}$ yields $\frac{Ax^2+Bx-C^2x^2}{\sqrt{Ax^2+Bx}+Cx}$. Because this limit exists, the degree of the numerator mst not exceed the degree of the denominator, so in order for this to be the case, $A - C^2 = 0$ so $A = C^2$. Substituting C^2 for A yields $\frac{\lim_{x \to \infty} Bx}{\sqrt{C^2x^2+2Bx}+Cx=2}$. Thus, $\frac{B}{2C} = 2$. The problem asks for $\frac{BC}{A}$. If we substitute C^2 for A again, we see that $\frac{BC}{A} = \frac{B}{C} = 2\frac{B}{2C} = 4$.

12. 20 - Let x be the distance along the shore that Ray paddles his boat. Then, the distance he must walk is $2 - x$ and the actual distance in the river he must paddle is $\sqrt{1+x}$. The time it takes to go a distance d at a rate r is $\frac{d}{r}$, so the total time, T, that Ray takes to get home is given by $T(x) = \frac{\sqrt{1+x}}{6} + \frac{2-x}{10}$. To find the value for x that optimizes $T(x)$, we set $T'(x) = 0$ and solve for x. So, $\frac{x}{6\sqrt{1+x^2}} - \frac{1}{10} = 0$, thus $\frac{x}{\sqrt{1+x^2}} = \frac{6}{10}$. With a bit of algebra, we arrive at $\frac{x=3}{4}$. Plugging back into T, we see that $T(\frac{3}{4}) = \frac{1}{3}$. Recall, however, that the speeds were given in hours, so the result of $\frac{1}{3}$ is also in hours. Converting to minutes yields the final answer of 20.

13. $3\arctan(\frac{x+3}{4}) + \ln|\frac{x-9}{x+9}| + C$ - $A = \int \frac{dx}{(x+3)^2+4^2} = \arctan(\frac{x+3}{4}) + C$. We use partial fractions to simplify to $B = \frac{1}{12}\int(\frac{1}{x-9} - \frac{1}{x+3})\,dx = \frac{1}{12}\ln(\frac{x-9}{x+3}) + C$ Therefore. $12(A+B) = 3\arctan(\frac{x+3}{4}) + \ln|\frac{x-9}{x+9}| + C$.

14. -4 - Terms for the expansion of $(x+y)^{-2}$ are generated using the following algorithm. The first term is x^{-2}. Each term is generated from the previous term by decrementing the x coefficient and incrementing the y coefficient. Next, the coefficient is found by multiplying the coefficient of the previous term by the x exponent of the previous term, then dividing the product by the y exponent of the new term. For example, if one term is Cx^my^n then the next term will be $C \cdot \frac{m}{n+1}x^{m-1}y^{n+1}$. Therefore, the expansion of $(x+y)^{-2}$ is $x^{-2} + (1 \cdot -2/1)x^{-3}y + (-2 \cdot -3/2)x^{-4}y^2 + (3 \cdot -4/3)x^{-5}y^3 + ...$ So the desired coefficient is -4.

15. $\frac{\sqrt{\pi}}{2}$ - This definite integral can be found in terms of another one whose value is known. With a u-substitution of $u = \frac{x}{\sqrt{2}}$, we see that $\int_0^\infty e^{-x^2}\,dx = \int_0^\infty \frac{\sqrt{2}}{2}e^{\frac{-u^2}{2}}\,du = \sqrt{\pi}\cdot\int_0^\infty \frac{1}{\sqrt{2\pi}}e^{\frac{-u^2}{2}}\,du = \sqrt{\pi}\cdot\frac{1}{2} = \frac{\sqrt{\pi}}{2}$. The final integral is that of the normal curve. It is given that the area under the normal curve on $(-\infty, \infty)$ is 1, and it is an odd function, so the area under half of it is $\frac{1}{2}$. So the final answer is $\frac{\sqrt{\pi}}{2}$.