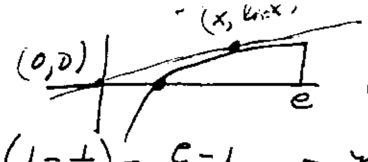
Team CAK – Armwood '96

1. $A=\int_1^e \ln x\,dx=(x\ln x-x)\Big|_1^e=(e-e)-(0-1)=1$



$\frac{\ln x-0}{x-0}=\frac{1}{x}$; $\ln x=1$; $x=e$; $m=1/e$

$B=\frac{A}{e-1}=\frac{1}{e-1}$; $C=1/e$; $B(A-C)=\frac{1}{e-1}\left(1-\frac{1}{e}\right)=\frac{e-1}{e(e-1)}=\boxed{1/e}$

2. $6x+4x^2\frac{dy}{dx}+8xy+2xy\frac{dy}{dx}+y^2=0$; $6+4\frac{dy}{dx}+8+2\frac{dy}{dx}+1=0$; $6\frac{dy}{dx}=-15$, $\frac{dy}{dx}$

$y=mx+b$; $1=-5/2+b$; $b=\boxed{7/2}$

3.  $2\int_0^4\sqrt{y}\,dy=4/3\,y^{3/2}\Big|_0^4=4/3\cdot 8=\frac{32}{3}$; $2\int_0^c\sqrt{y}\,dy=\frac{16}{3}$; $\frac{4}{3}c^{3/2}=\frac{16}{3}$; $c^{3/2}=4$, $\boxed{c= \dots \text{ or}}$

4. $\frac{dy}{dx}=\frac{e^x}{e^y}$; $e^y dy=e^x dx$; $e^y+C=e^x$; $2+C=1$; $C=-1$; $e^y=e^x+1$; $\boxed{y=\ln(e^x+1)}$

5. $\int_0^{t^2}e^{-u}du=-e^{-u}\Big|_0^{t^2}=-e^{-t^2}+1$; $f(x)=\int_0^x(-e^{-t^2}+1)dt$; $f'(x)=-e^{-x^2}+1$; $f'(2)=-e^{-4}+1$; $\boxed{1-e^{-4}}$

6. $A=\lim_{x\to1}\frac{\ln x-x+1}{x\ln x-\ln x}=\lim_{x\to1}\frac{\left(\frac{1}{x}-1\right)x}{\left(1+\ln x-\frac{1}{x}\right)x}=\lim_{x\to1}\frac{1-x}{x+x\ln x-1}=\lim_{x\to1}\frac{-1}{1+1+\ln x}=-1/2$

$B=\frac{1}{4}$ circle of radius $3=9\pi/4$; $C=f'(4)=\frac{1}{2\sqrt4}=\frac{1}{4}$; $AB/C=-\frac{9\pi}{8}\cdot4=\boxed{-9 \dots}$

7. $f(x)=\frac{1}{2}[6x\ln e+\ln\sin 4x]$; $f'(x)=\frac{1}{2}[6+4\cot 4x]=3+2\cot 2\pi/9=\boxed{3 \dots}$

8. $x^5-2x^3-7x-8=-6$; $x^5-2x^3-7x-2=0$

$2)\ \ 1\ \ 0\ \ -2\ \ 0\ \ -7\ \ -2$
$\quad\ \ \ \ \ 2\ \ \ 4\ \ \ 4\ \ \ 8\ \ \ 2$
$\quad\ 1\ \ 2\ \ \ 2\ \ \ 4\ \ \ 1\ \ \ 0$

any other real roots must be negative.

$f'(x)=5x^4-6x^2-7$
$f'(2)=80-24-7=49$

9. $\int_0^{\pi/6}\sin(\sin u)\cos u\,du$ Let $w=\sin u$, $dw=\cos u\,du$; $\int_0^{1/2}\sin w\,dw=-\cos w\Big|_0^{1/2}=\boxed{1-\cos\frac{1}{2}}$

10.

15. $4x+14y\frac{dy}{dx}=0$; $\frac{dy}{dx}=-\frac{2x}{7y}=-\frac{4}{7}$; $y-1=-4/7(x-2)$; $-1=-4/7x+8/7$; $-7=-4x+8$ $\boxed{x= \dots}$

11. $2ax+1=0$ at $x=1/2$; $a+1=0$ and $a=-1$. $\int_{-1}^1(-x^2+x+c)dx=\left(-\frac{1}{3}x^3+\frac{1}{2}x^2+cx\right)\Big|_{-1}^1=-\frac{2}{3}+2c$

$2c=2/3$; $c=1/3$

12. $\int_0^1\frac{1}{\sqrt{x}+1}dx=\int_1^2\frac{2u-2}{u}du=\int_1^2\left(2-\frac{2}{u}\right)du=(2u-2\ln u)\Big|_1^2=4-2\ln2-(2-0)=\boxed{2-2 \dots}$

Let $u=\sqrt{x}+1$; $(u-1)^2=x$; $2(u-1)du=dx$ $\boxed{\text{or } 2-\ln \dots}$

13. $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}=\sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)=\frac{1}{2}\lim_{n\to\infty}\left[\left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\cdots+\cancel{\frac{1}{2n-1}}+ \dots\right]$

$=\boxed{1/2}$

14. $\lim_{x\to0}\frac{\int_0^{x^2}\sqrt{1+\sin t}\,dt}{x\cos x}=\lim_{x\to0}\frac{\left(\sqrt{1+\sin x^2}\right)(2x)}{-x\sin x+\cos x}=\boxed{0}$

10. $\int_1^2\left(\sqrt{y}-\frac{y}{3}\right)dy=\left(\frac{2}{3}y^{3/2}-\frac{1}{6}y^2\right)\Big|_1^2=\frac{4}{3}\sqrt2-\frac{2}{3}-\left(\frac{2}{3}-\frac{1}{6}\right)=$

15. (Above #11)

$\frac{4\sqrt2}{3}-\frac{7}{6}$