Armwood

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$\int \sin u\,du = -\cos u$ $\quad \int -\cos u = -\sin u$ $\quad \int -\sin u = \cos u$ $\quad \int \cos u\,du = \sin u$

1. since each time $u = 2x$ the tenth is $-\sin(2x)$ 1024

2. $A = \frac{1}{4}\int_0^4 (x^2-4x+5)\,dx = \frac{1}{4}\Big|_0^4 \frac{x^3}{3} - 2x^2 + 5x = \frac{1}{4}\left[\frac{64}{3} - 32 + 20\right] = \frac{1}{4}\left[\frac{64}{3} - \frac{36}{3}\right] = \frac{1}{4}\cdot\frac{28}{3} = \frac{7}{3}$

B $\frac{f(4)-f(0)}{4} = 2x-4$ $\quad \frac{5-5}{4} = 2x-4$ $\quad 2x = 4$, $x = 2$ $\quad$ C $\quad 2x-4=0$, $x=2$ $\quad$ D max at $x=0$ or $x=4$ $\quad 1, 5$ $\quad \frac{7}{3}, 2, 2, 5$

$\boxed{\frac{140}{3}}$

3. $\int_0^8 \frac{1}{x+1}\,dx = \ln 9 - \ln 1 = \ln 9$ $\quad \int_0^k \frac{1}{x+1}\,dx = \ln(k+1) - \ln 1 = \ln(k+1)$

$\ln(k+1) = \frac{1}{2}\ln 9$ $\quad \ln(k+1) = \ln 3$ $\quad k = 2$

4. $f'(x) = \frac{e^x x^2 - 2xe^x}{x^4} = \frac{e^x(x-2)}{x^3}$ $\quad$ min occurs at $x = 2$, $y = \frac{e^2}{4}$ $\quad \boxed{\left(2, \frac{e^2}{4}\right)}$

5. $\ln y = \ln e^{x\sin x} = \sin x \ln e^x = x\sin x$ $\quad \left[\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right] e^{\frac{\pi}{2}\sin\frac{\pi}{2}} = \boxed{e^{\frac{\pi}{2}}}$

$\frac{y'}{y} = x\cos x + \sin x$ $\quad y' = [x\cos x + \sin x]\,y$

6. $x(t) = \sin t + t + 1$ $\quad a(t) = -\sin t$ $\quad v\left(\frac{2\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) + 1 = \frac{1}{2}$

$x(\pi/6) = \sin\frac{\pi}{6} + \frac{\pi}{6} + 1 = \frac{\pi}{6} + \frac{3}{2}$ $\quad a\left(\frac{5\pi}{6}\right) = -\frac{1}{2}$ $\quad \cos t = -1 \quad t = \pi$

$x(0) = 1$ $\quad x(\pi) = \pi + 1$ $\quad x(2\pi) = 2\pi + 1$ $\quad \frac{\pi}{6} + \frac{3}{2} - \frac{1}{2} + \frac{1}{2} + 2\pi$ $\quad \boxed{\frac{13\pi}{6} + \frac{3}{2} \text{ or } \frac{13\pi + 9}{6}}$

$x(\pi) - x(0) = \pi$ $\quad x(2\pi) - x(\pi) = \pi$ $\quad \therefore 2\pi$

7. $4x + y = 96$ $\quad V = x^2(96 - 4x)$ $\quad V'(x) = 192x - 12x^2$

$y = 96 - 4x$ $\quad V = 96x^2 - 4x^3$ $\quad 12x(16 - x)$ $\quad 256(32) = 8192$

8. $\lim_{x\to 0}\left(\frac{1}{7x^2} - \frac{\sin x}{7x^2\tan x}\right)$ $\quad \lim_{x\to 0}\left(\frac{1}{7x^2} - \frac{\cos x}{7x^2}\right) = \lim_{x\to 0}\frac{\sin x}{14x} = \lim_{x\to 0}\frac{\cos x}{14} = \frac{1}{14}$

9. $f(f(x)) = [f(x)]^5 + [f(x)]^4 + [f(x)]^3 + [f(x)]^2 + f(x) + 1$

$dy\,[f(f(x))] = \left(5[f(x)]^4 f'(x) + 4[f(x)]^3 f'(x) + 3[f(x)]^2 f'(x) + 2[f(x)]\cdot f'(x) + f'(x)\right)dx$

$dy = f'(x)\left[5[f(x)]^4 + 4[f(x)]^3 + 3[f(x)]^2 + 2[f(x)] + 1\right]dx$ $\quad f(0) = 1$, $f'(0) = 1$

$dy = 1[5\cdot 1 + 4\cdot 1 + 3\cdot 1 + 2\cdot 1 + 1](.01) = 15(.01) = .15$

10. $10x + 2x^2y' + 4xy + 2yy' = 0$ $\quad \boxed{\text{at } x = 2}$ $\quad 20 + 8y + y^2 = 8$ $\quad y = -6$

$20 + 8y' + 8y + 2yy' = 0$ $\quad y^2 + 8y + 12 = 0$ $\quad (y+6)(y+2) = 0$ $\quad y = -2$

$y' = \frac{-20 - 8y}{2y + 8}$ $\quad$ at $y = -2$, $m_{tan} = -1$ $\quad$ at $y = -6$, $m_{tan} = -7$ $\quad -1 \times -7 = 7$

11. $\frac{dy}{y} = \cos x\,dx$ $\quad \ln|y| = \sin x + C$ $\quad \ln 1 = \sin 0 + C$ $\quad C = 0$ $\quad \ln|y| = \sin x$ $\quad -1 = \sin x$ $\quad x = \frac{3\pi}{2}$

12. $\int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx + \int_0^1 \frac{3\,dx}{\sqrt{4-x^2}}$ $\quad 3\int_0^1 \frac{dx}{\sqrt{4-x^2}} = 3\Big|_0^1 \arcsin\frac{x}{2} = 3\left[\frac{\pi}{6} - 0\right] = \frac{\pi}{2}$

$u = 4 - x^2$ $\quad du = -2x\,dx$

$-\frac{1}{2}\int_4^3 u^{-1/2}\,du = -\Big|_4^3 \sqrt{u} = -(\sqrt{3} - 2) = 2 - \sqrt{3}$ $\quad \boxed{2 - \sqrt{3} + \frac{\pi}{2} \text{ or } \frac{4 - 2\sqrt{3} + \pi}{2}}$

13. $2\pi\int_0^4 \left(\sqrt{25 - x^2}\right)^2 - (3)^2 = 2\pi\int_0^4 (16 - x^2)\,dx = 2\pi\Big|_0^4 16x - \frac{x^3}{3} =$

$2\pi\left(64 - \frac{64}{3}\right) = 2\pi\left[\frac{192 - 64}{3}\right] = \frac{2\pi(128)}{3} = \frac{256\pi}{3}$

14. $2\pi\int_0^{\frac{\pi}{4}} x\sin x\,dx$ $\quad u = x$, $du = dx$ $\quad v = -\cos x$, $dv = \sin x$ $\quad -x\cos x + \int \cos x\,dx$ $\quad 2\pi\Big|_0^{\frac{\pi}{4}} -x\cos x + \sin x =$

$2\pi\left[\frac{-\pi}{4}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right] = \frac{-\sqrt{2}\pi^2}{4} + \sqrt{2}\pi$ or $\sqrt{2}\pi\left(\frac{-\pi}{4} + 1\right)$ or $\frac{-\sqrt{2}\pi^2 + 4\sqrt{2}\pi}{4}$

15. [illegible]

$t = 0$, $y = 10$ $\quad t = ?$
$t = 20$, $y = 6$ $\quad y = 4$

$\frac{dy}{dt} = ky$ $\quad \frac{dy}{y} = k\,dt$ $\quad \ln|y| = kt + C$

$\ln y = kt + \ln 10$
$\ln 6 = 20k + \ln 10$
$\ln 6 - \ln 10 = 20k$
$\frac{\ln .6}{20} = k$

$\ln y = \frac{\ln(.6)}{20}t + \ln 10$

$\ln 4 = \frac{\ln(.60)}{20}t + \ln 10$ $\quad \ln(.4) = \frac{\ln(.6)}{20}t$ $\quad t = \frac{20\ln(.4)}{\ln(.6)} \approx$ 36 minutes