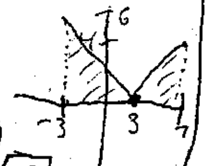
Solutions

3/8/03 Calc. Sol. Indiv.

① $A = \frac{1}{2}(6)(6) + \frac{1}{2}(4)(4)$

$A = 26$ $\boxed{C}$



② $Avg = \frac{1}{\pi/2}\int_0^{\pi/2} \sin x = \frac{2}{\pi}\cdot 1 = \frac{2}{\pi}$ $\boxed{D}$

③ If Y = revenue then

$Y = Pq = P(30000 - 40P) = 30{,}000P - 40P^2$

$Y' = 30{,}000 - 80P = 0 \Rightarrow P = 375$ $\boxed{E}$

④ $x^2 + y^2 = 9 \Rightarrow y = \sqrt{9 - x^2}$

$\frac{dy}{dx} = \frac{-x}{\sqrt{9-x^2}}\Big|_{x = \frac{3\sqrt5}{5}} = \frac{-3\sqrt5/5}{\sqrt{36/5}} = -\frac{1}{2}$ $\boxed{C}$

⑤ $h'(x) = g'(f(x))\cdot f'(x)$

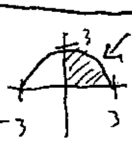
$h'(1) = g'(2)\cdot -5 = 3\cdot -5 = -15$ $\boxed{D}$

⑥ The degree of the denominator is greater, implying that the limit will approach 0. $\boxed{C}$

⑦ I - false: f is undefined at $x = 5$.

II - True

III - false, see I. $\boxed{B}$

⑧ 

$\sqrt{(3+t)(3-t)}$

$= \frac{1}{4}\pi(3)^2 = \frac{9\pi}{4}$ $\boxed{A}$

⑨ 10 min = 600 sec.

$s = \int_0^{600}\sqrt{\frac{x}{15}}\,dx$

$s = \frac{\sqrt{15}}{15}\int_0^{600} x^{1/2}\,dx = \frac{\sqrt{15}}{15}\left(\frac{2}{3}x^{3/2}\Big|_0^{600}\right)$

$= \frac{2\sqrt{15}}{45}\left(x^{3/2}\Big|_0^{600}\right) = \frac{2\sqrt{15}}{45}(6000\sqrt6)$

$= 800\sqrt{10}$ $\boxed{A}$

⑩ $\int \frac{x}{x^2+3}\,dx$, $u = x^2 + 3$, $du = 2x\,dx$

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|x^2+3| + C$ $\boxed{A}$

⑪ $v(t) = 3t^2 - 6t - 36 = 0$

$t^2 - 2t - 12 = 0$

$t = 1 + \sqrt{13}$ or $\cancel{t = 1 - \sqrt{13}}$

≈ 4.6 $\quad \cancel{\approx -2.6}$

$t = 1 + \sqrt{13}$ $\boxed{C}$

⑫ $\sum_{x=0}^{\infty}(2/3)^x = 1 + \sum_{x=1}^{\infty}(2/3)^x$

$= 1 + \frac{2/3}{1 - 2/3} = 1 + 2 = 3$ $\boxed{C}$

⑬ $f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$1999 \equiv 3 \pmod 4 \Rightarrow f^{(1999)}(x) = f^{(3)}(x) = -\cos x$ $\boxed{D}$

⑭ $y'(x) = x^2 - 7x + 10 = 0$

$\Rightarrow x = 5$ or $x = 2$

$y(0) = 0$, $y(2) = \frac{26}{3}$, $y(5) = \frac{25}{6}$,

$y(7) = \frac{77}{6}$ $\boxed{C}$

⑮ $\lim_{x\to\infty} s(x) = 2x|_{x=1} = 2$, must be true by the Sandwich Theorem $\boxed{C}$

⑯ $\frac{dx}{dy} = \frac{x-2}{y} \Rightarrow \int\frac{1}{x-2}\,dx = \int\frac{1}{y}\,dy$

$= \ln|x-2| = \ln|y|$

$\Rightarrow y = |x-2|$, $x(0) = 2$ $\boxed{C}$

⑰ $\frac{d}{dx}(a^u) = a^u \ln a \cdot du$

$\Rightarrow \frac{dy}{dx} = 3^{x^2}\ln(3)\,2x$ $\boxed{D}$

⑱ $f(x) = \sqrt{x}$, $f'(3) = \frac{1}{2\sqrt3} = \frac{\sqrt3}{6}$ $\boxed{B}$

⑲ $\frac{dy}{dx} = 2\sin x\cos x = \sin 2x$

$\sin\left(2\frac{\pi}{8}\right) = \sin\frac{\pi}{4} = \frac{\sqrt2}{2}$ $\boxed{B}$

⑳ $v(t) = 2t^2 - t + 14 = 0$ false.

$\Rightarrow$ Distance $= \int_0^5 2t^2 - t + 14$

$= \frac{2t^3}{3} - \frac{t^2}{2} + 14t\Big|_0^5 = \frac{845}{6}$ $\boxed{E}$

㉑ $x^2y + y^2x = 2$

$\Rightarrow x^2\frac{dy}{dx} + 2xy + y^2 + 2xy\frac{dy}{dx} = 0$

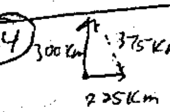
$\Rightarrow \frac{dy}{dx} = \frac{-(2xy + y^2)}{x^2 + 2xy}$ at $(1,1)$ $= \frac{-(2+1)}{1+2} = \frac{-3}{3} = -1$ $\boxed{A}$

㉒ $\int\sin(2x)\,dx = \int 2\sin x\cos x = \sin^2 x + C$ $\boxed{A}$

㉓ $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

288π in$^3 = \frac{\pi}{6}$ ft$^3 = \frac{4}{3}\pi r^3 \Rightarrow r = \frac{1}{2}$ ft

$\frac{\pi}{6}\frac{ft^3}{sec} = 4\pi\left(\frac{1}{2}\right)^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{6}$ ft/sec $= 2$ inches/sec $\boxed{A}$

㉔ 

$c^2 = a^2 + b^2$

$\cancel{2}c\frac{dc}{dt} = \cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt}$

$\frac{dc}{dt} = \frac{(300\cdot100) + (225\cdot75)}{375} = 125\ \frac{km}{hr}$ $\boxed{B}$

㉕ 

$V = s(3-2s)(2-2s) = 4s^3 - 10s^2 + 6s$

$V' = 12s^2 - 20s + 6 = 0$

$\Rightarrow s = 1.274$ or $s = 0.392$

$\cancel{V(1.274) = -.316}$ $\quad V(.392) = 1.056$

$\ln(.392) = -0.936$ $\boxed{D}$

㉖ $\int\frac{9-x}{x^2+10x+21} = \int\frac{9-x}{(x+3)(x+7)} = \int\frac{3}{x+3} + \int\frac{-4}{x+7}$

$= 3\ln|x+3| - 4\ln|x+7| + C$ $\boxed{B}$

㉗ $\int_0^1 x^{3/n} = \frac{n}{3+n}x^{\frac{3+n}{n}}\Big|_0^1 = \frac{n}{3+n}(1) - \frac{n}{3+n}(0) = \frac{n}{3+n}$

$\frac{n}{3+n} = \frac{8}{11} \Rightarrow n = 8$ $\ln(8) = 2.07$ $\boxed{E}$

㉘ $\int_0^{\pi/3}\sec x\tan x\,dx = \int_0^{\pi/3}\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}dx$ $\int_0^{\pi/3}\frac{\sin x}{\cos^2 x}dx$

$= \int_0^{\pi/3}\sin x\cdot\sec^2 x\,dx$, $u = \sin x$, $du = \cos x\,dx$, $v = \tan x$, $dv = \sec^2 x\,dx$

$= \sin x\tan x - \int\tan x\cos x\,dx = \sin x\tan x + \cos x\Big|_0^{\pi/3} = 1$ $\boxed{B}$

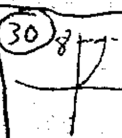
㉙ $2\sin t = \cos(2t) = 2\cos^2 t - 1 = 2(1 - \sin^2 t) - 1$

$2\sin t = 2 - 2\sin^2 t - 1 \Rightarrow 2\sin^2 t + 2\sin t - 1 = 0$

$2y^2 + 2y - 1 = 0 \Rightarrow y = .366$ or $\cancel{y = -1.367}$

$\sin t = .366$ or $\cancel{\sin t = -1.367}$

$t = 0.375$ $\boxed{B}$

㉚ 

$V = \pi\int_0^8 (x^{1/3})^2\,dx = \pi\int_0^8 x^{2/3}\,dx$

$= \pi\left[\frac{3}{5}x^{5/3}\Big|_0^8\right] = \frac{96\pi}{5}$ $\boxed{C}$

Calculus Solutions Individual 3/8/03 MHS