Calculus Ind. Solutions – March '98

1. A $y = e^{\sin\sqrt{x}} \Rightarrow y' = e^{\sin\sqrt{x}} \cdot \cos\sqrt{x} \cdot \frac{1}{2} x^{-1/2} = e^{\sin\sqrt{x}} \frac{\cos\sqrt{x}}{2\sqrt{x}}$

2. A Use fint((x-1)^2+1), x, 1, 2) on calc.

3. D $a = -32 = \frac{dv}{dt} \Rightarrow v = -32t + c \ (c = 96) \Rightarrow s = -16t^2 + 96t$, max @ 144.

4. D $\lim_{x \to 0} f(x) = \frac{c}{0} \Rightarrow$ No Horiz Asy.

5. C

6. B $y' = 2\cos x$ @ $x = \pi/3 = 1$. If $x = \pi/3$, $y = \sqrt{3} \Rightarrow y - \sqrt{3} = 1(x - \pi/3)$ or $3x - 3y = \pi - 3\sqrt{3}$

7. A $y'' = x + 2 \Rightarrow y' = \frac{x^2}{2} + 2x + c_1 \ (c_1 = 3) \Rightarrow y = \frac{x^3}{6} + x^2 + 3x + c_2$

8. A $P = \frac{50t^3}{3} - 40t^{3/2} + c \ (c = 25000) \Rightarrow 50{,}000 = \frac{50t^3}{3} - 40t^{3/2} + 25000$ $(c_2 = 1)$

9. A A flat place occurs where the elev. is neither inc or dec. or when $dy/dx = 4(3x^2 - 24x + 48) = 0 \Rightarrow x = 4$ INTERSECTION @ 11.6

10. C $dy = \frac{3}{\sqrt{9 - x^2}} dx = \frac{dx}{\sqrt{1 - (\frac{x}{3})^2}} \Rightarrow y = 3 \arcsin\frac{x}{3} + c$

11. D Dimensions are x by $\frac{3000}{x}$. Cost of shrubs $= 25(2x + \frac{3000}{x})$; Cost of fence $= 20(\frac{300}{x},$
$TC = 50x + \frac{75000}{x} + \frac{60000}{x} = 50x + \frac{135000}{x} \Rightarrow C' = 50 - \frac{135000}{x^2} = 0$ at $x = 52'$.

12. D Let $u = \ln x^3$, $dv = dx$; $du = \frac{3}{x}$, $v = x$ $\Rightarrow \int \ln x^3 dx = x \ln x^3 - \int 3 dx = 3x \ln x - 3x + c$

13. A $\frac{dV}{dt} = \frac{1}{12\pi}(2Ch\frac{dC}{dt} + C^2\frac{dh}{dt}) \Rightarrow$ At $C = 5$, $\frac{dC}{dt} = .2$, $h = 22$, $\frac{dh}{dt} = 4 \Rightarrow \frac{dV}{dt} \approx 3.82 \text{ ft}^3/\text{hr}$.

14. B Ratio Test: $\lim_{n \to \infty} \left| \frac{(x+5)^{n+1}}{6^{n+1}} \cdot \frac{6^n}{(x+5)^n} \right| = \left| \frac{x+5}{6} \right| < 1 \Rightarrow -11 < x < 1$

15. C $f_x(x, y)$ holds y as a constant $\Rightarrow = \frac{1}{x} + (e^{xy})y$

16. E $A = \int_0^{\pi} \frac{1}{2}(2(\cos\theta + \sin\theta))^2 d\theta = 2\pi$

17. C $v = -24t + 51$ @ $t = 1 \Rightarrow v = 27$

18. C $y' = -2x \Rightarrow (y')^2 = 4x^2$; $SA = 2\pi \int_0^1 x\sqrt{1+4x^2}\,dx$

19. E None are always true of $\ln(\ln x)$.

20. D $dy = 4x^{-1/2}\,dx \Rightarrow y = 8x^{1/2} + c \;(c=4) \Rightarrow y = 8x^{1/2} + 4$

21. B Ratio Test: $\lim_{n\to\infty} \left| \frac{3^{n+1}(x-2)^{2n+3}}{(n+1)!} \cdot \frac{n!}{3^n (x-2)^{2n+1}} \right| = \lim_{n\to\infty} \frac{3}{n+1} |x-2| = 0$

$\therefore$ radius of conv $= \infty$ (conv for all x)

22. C $\lim_{x\to \pi/2} (\tan x)^{\cos x} = y \Rightarrow \ln y = \lim_{x\to\pi/2} \cos x (\ln(\tan x)) = \lim_{x\to\pi/2} \frac{\ln(\tan x)}{\sec x}$

Using L'Hopital's $\Rightarrow \ln y = \lim_{x\to\pi/2} \frac{\cos x}{\sin^2 x} = 0$

$e^0 = y \Rightarrow y = 1$

23. D $\lim_{a\to\infty} \int_0^a \frac{8}{x^2+4}\,dx = 4 \lim_{a\to\infty} \arctan\frac{x}{2}\Big]_0^a = 2\pi$

24. A $V = \int_{-2}^{2} \frac{1}{2}\pi \left(\frac{4-x^2}{x}\right) dx = \frac{4\pi}{3}$

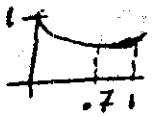
25. A $e^u = 1 + u + \frac{u^2}{2} \ldots \Rightarrow e^{-x^2} \approx 1 - x^2 + \frac{x^4}{2}$

$\int_0^1 e^{-x^2}\,dx \approx \int_0^1 dx - \int_0^1 x^2\,dx + \int_0^1 \frac{x^4}{2}\,dx \approx 0.767$

26. A $d = \int_0^5 (80 - 16t)\,dt = 200'$

27. B Avg Value $= \frac{x(\pi/2) - x(0)}{\pi/2 - 0} = \frac{(\pi^2+7) - 0}{\pi/2} \approx 10.74$

28. B A strip of the city, of width Δr, $\perp$ to the road at a distance r from it has length $\sqrt{25-r^2}$, hence area $\sqrt{25-r^2}\,\Delta r$. $\therefore$ population in the strip is $(20-4r)\sqrt{25-r^2}\,\Delta r$. The integral is $P = 2\int_0^5 (20-4r)\sqrt{25-r^2}\,dr \approx 226{,}000$

29. C The curve looks like 

. Notice the solution curve starts increasing at about $x = .7$. At $x=1$, $y \approx 0.75$.

30. B Amt dumped $= \int_0^{52\text{ weeks}} (\text{rate})(\text{time}) = \int_0^{52} \frac{t}{60} t^{2/3}\,dt \approx 7.24$ tons

Calculus Individual Test — March Regional, 1998

Answers

1. A
2. A
3. D
4. D
5. C

6. B
7. A
8. A
9. A
10. C

11. D
12. D
13. A
14. B
15. C

16. E (2π)
17. C
18. C
19. E
20. D

21. B
22. C
23. D
24. A
25. A

26. A
27. B
28. B
29. C
30. B