

Notes: NOTA represents "None of the Above" and is choice E on each question.
Approximate answers are given to the nearest thousandth unless otherwise stated.

1. Which of the following are properties of the graph of $y = \sqrt[3]{x^2 - 4}$?

- I. It is symmetric with respect to the y -axis.
- II. The range is $y \geq -1.5$.
- III. The domain is all real numbers.

a. I, II, and III b. I and II only c. I and III only d. II and III only e. NOTA

2. Which of the following is the solution set of $x^3 - 4x^2 + 3.99x > 0$?

a. $(0, 1.9) \cup (2.1, \infty)$ b. $(0, \infty)$ c. $(-\infty, 0)$ d. $(-\infty, 0) \cup (1.9, 2.1)$ e. NOTA

3. Evaluate $\lim_{x \rightarrow 0} \frac{(.1+x)^2 - .01}{x}$

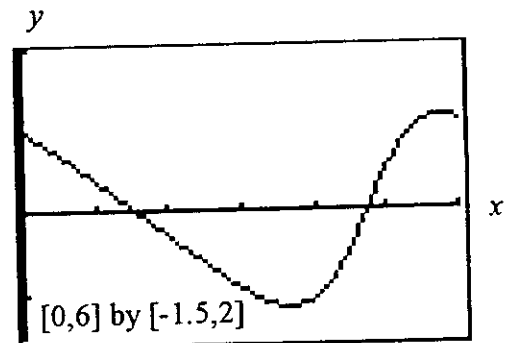
a. 0 b. .1 c. .2 d. .21 e. NOTA

4. A function $y = f(x)$ is known to be negative at $x = 0$ and positive at $x = 1$. Which of the following is a property of function f ?

- a. the equation $f(x) = 0$ has a solution between 0 and 1.
- b. f is strictly increasing between 0 and 1.
- c. f is concave up between 0 and 1.
- d. f is differentiable on $(0, 1)$.
- e. NOTA

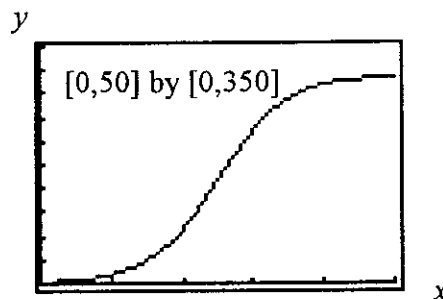
5. The following is the graph of the derivative of a function f on $[0, 6]$. Which of the following is closest to the maximum value of f ?

- a. $f(0)$
- b. $f(1.5)$
- c. $f(3.8)$
- d. $f(6)$
- e. NOTA



6. The following graph represents the graph of a fruit fly population over a period of 50 days. On approximately which day is the population growing the fastest?

- a. day 20
- b. day 23
- c. day 35
- d. day 50
- e. NOTA



7. On which intervals is the graph of $y = \frac{5x}{x^2 + 4}$ concave up?

- a. $(-3.5, 0) \cup (3.5, \infty)$
- b. $(-\infty, -3.5) \cup (0, 3.5)$
- c. $(-\infty, 0)$
- d. $(-2\sqrt{3}, 0) \cup (2\sqrt{3}, \infty)$
- e. NOTA

8. Which of the following is the equation of one of the tangents to $y = x^3 + 3$ where the slope is 12?

- a. $12x - y = 11$
- b. $y = 12x + 19$
- c. $12x - y = 19$
- d. $x + 12y = -19$
- e. NOTA

9. What is the smallest slope of the graph of $y = x^3 - 3x + 1$ on $[-2, 2]$?

- a. -12
- b. -2
- c. -1
- d. 0
- e. NOTA

10. Find the sum of the slopes of the tangent and normal lines to the graph of $x^2 - xy + y^2 = 7$ at $(-1, 2)$.

- a. -1
- b. $-\frac{11}{20}$
- c. $-\frac{9}{20}$
- d. 0
- e. NOTA

11. Use a differential to approximate the change in the volume of a cube with edge of length s when the edge lengths change from x_1 to $x_1 + dx$.

- a. $(dx)^3$
- b. $(3s^2)ds$
- c. $x_1^3 + 3x_1^2 dx$
- d. $3x_1^2 dx$
- e. NOTA

12. A point in the domain of a function f at which $f' = 0$ or f' does not exist is a(n)

- a. local (relative) extremum
- b. critical point
- c. inflection point
- d. absolute extremum
- e. NOTA

13. For $f(x) = x \sin x$ on $[-4, 4]$, which of the following are closest to all values of c that satisfy the Mean Value Theorem (for derivatives)?

- a. $\pm 2.029, 0$
- b. $\pm 2, 0$
- c. $0, \pm \pi$
- d. $-1.708, -.399, 2.290$
- e. NOTA

14. A particle moving on the x -axis is located at s where $s(t) = 2t^3 - 13t^2 + 22t - 5$ on $[1, 3.5]$. What is its maximum speed and what direction is it headed?

- a. 13/6; left b. 107/54; right c. 3.18; left d. 1.15; right e. NOTA

15. A right triangle whose hypotenuse is $\sqrt{3}$ meters long is revolved about one of its legs to generate a right circular cone. Find the sum of the radius and height of the cone of greatest volume that can be made this way.

- a. 1 b. $1 + \sqrt{2}$ c. $1 + \sqrt{3}$ d. 3 e. NOTA

16. Use one iteration of Newton's Method to approximate the zero of $y = \tan x$ using an initial approximation of 3.

- a. 3.143 b. 3.142 c. 3.141 d. 3.140 e. NOTA

17. You are designing a 1000 cm^3 right circular cylindrical can whose manufacture will take waste into account. There is no waste in material used for the sides, but tops and bottoms of radius r cm will be cut from squares that measure $2r$ cm on a side. In determining the total aluminum used for each can you must include the waste for the tops and bottoms. Find the ratio of the height to the radius for the most economical cans.

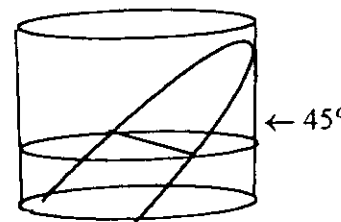
- a. 2 to 1 b. 6 to π c. 8 to π d. 9 to π e. NOTA

18. In a cylindrical tank, the rate at which the water level drops is proportional to the square root of the water's depth. Find an equation that relates the water level, y , to time, t , using k as a constant of proportionality. The water level is 1 at $t = 0$.

- a. $y = \left(\frac{k}{2}t + 1\right)^2$ b. $y = \frac{k}{2}t^2 + 1$ c. $y = \frac{k}{2}\sqrt{t} + 1$ d. $y = \left(\frac{k}{2}\sqrt{t} + 1\right)^2$ e. NOTA

19. A curved wedge is cut from a cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.

- a. 16/3
b. 64/3
c. 128/3
d. 43
e. NOTA



20. Find the area between the curve $x = (y - 1)^2$ and the line $y = x - 1$.

- a. $13/3$ b. $16/3$ c. $32/3$ d. $33/2$ e. NOTA

21. Let $F(x) = \int_1^x \frac{\cos t}{t} dt; x > 0$ and $G(x) = \int_{.5}^x \frac{\cos t}{t} dt; x > 0$. Which of the following are true?

- I. $F(x) - G(x) = C$ where C is a constant
 II. $F'(x) = G'(x)$ for all x in the domain
 III. $F(x) < G(x)$ for all x in the domain

- a. II only b. I and II only c. II and III only d. I, II, and III e. NOTA

22. Evaluate $\int \frac{\sin(4x - 1)}{1 - \sin^2(4x - 1)} dx$

- a. $\frac{1}{4} \sec(4x - 1) + C$
 b. $\frac{1}{4} \sin^{-1}(4x - 1) + C$
 c. $\frac{1}{4} \cos^{-1}(4x - 1) + C$
 d. $\frac{1}{4} \tan(4x - 1) + C$
 e. NOTA

23. Find the length of the curve $y = \ln(\cos x)$ between $x = 0$ and $x = \frac{\pi}{4}$.

- a. $\ln\left(\frac{\sqrt{2} + 2}{2}\right)$ b. $\sqrt{2} - 1$ c. $\ln(\sqrt{2} + 1)$ d. $\ln(\sqrt{2} - 1)$ e. NOTA

24. The model $\frac{dy}{dt} = ky(16 - y)$ can be used to describe current population growth. There were 2 billion people in 1925 and 4 billion people in 1975. Assume y is the population t years after 1925 and $y < 16$ billion. Find the growth constant, k .

- a. $\frac{1}{800} \ln \frac{7}{3}$ b. $\frac{8}{25} \ln 48$ c. $\frac{1}{25} \ln \frac{1}{3}$ d. $\frac{1}{800} \ln \frac{12}{5}$ e. NOTA

25. Evaluate $\lim_{x \rightarrow 0} \frac{\tan^{-1} x - x}{8x^3}$

- a. $-\frac{1}{25}$ b. $-\frac{1}{23}$ c. $-\frac{1}{20}$ d. $\frac{1}{24}$ e. NOTA

26. Evaluate $f'(x)$ if $f(x) = \cot(x \sin x)$

- a. $-\csc^2(x^2 \sin x \cos x)$
 b. $(x \cos x + \sin x) \csc^2(x \sin x)$
 c. $\frac{x \cos x + \sin x}{\cos^2(x \sin x) - 1}$
 d. $-x \cot x - \sec(x \sin x)$
 e. NOTA

27. If $y = x^y$, which of the following is an expression for $\frac{dy}{dx}$ if $x > 0$ and $y > 0$?

- a. $\frac{x^{2y-1}}{1 - \ln y}$ b. $y \left(\frac{1}{x} - \frac{1}{\ln y} \right)$ c. $\frac{y^2}{x - \ln x}$ d. $\frac{1 - y \ln x}{x}$ e. NOTA

28. Evaluate $\lim_{x \rightarrow b} \frac{f(x)}{2(x-b)} \int_b^x f(t) dt$ where f is a differentiable function.

- a. $\frac{f(b)}{2}$ b. $\frac{(f(b))^2}{2}$ c. $\frac{f(b) \cdot f'(b)}{2}$ d. 0 e. NOTA

29. Evaluate $\sum_{n=0}^{\infty} \left(\frac{1}{\sum_{k=0}^n \binom{n}{k}} \right)$

- a. 0 b. $\frac{1}{2}$ c. 1 d. 2 e. NOTA

30. Use the first two non-vanishing terms of the Maclaurin series to approximate $\sin \frac{1}{2}$ to the nearest ten-thousandth.

- a. .4794 b. .4793 c. .4792 d. .4791 e. NOTA