Atmwood '96 ... Calculus Endurance Solutions. 5/9/96

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1. I and III only $\boxed{C}$

2. $x(x^2-4x+3.99)>0$

$x(x-1.9)(x-2.1)>0$

$0<x<1.9$ or $x>2.1$ $\boxed{A}$

3. $=f'(.1)$ where
$f(x)=x^2 \quad f'(x)=2x$
$f'(.1)=.2$ $\boxed{C}$

4. $\boxed{E}$ It is not known that $f$ is continuous.

5. $f$ is increasing, then decreasing, then increasing. Clearly the maximum is $f(1.5)$ or $f(6)$ approximately.
$\int_0^3 f'(x)\,dx \doteq 0$ from the graph
and $\left|\int_3^{4.8} f'(x)\,dx\right| > \left|\int_{4.8}^{6} f'(x)\,dx\right|$
$\therefore f(6) < f(1.5)$ $\boxed{B}$

6. $\boxed{B}$

7. $y' = \frac{(x^2+4)5-5x(2x)}{(x^2+4)^2} = \frac{20-5x^2}{(x^2+4)^2}$

$y'' = 5\left[\frac{(x^2+4)^2(-2x)-(4-x^2)\cdot 2(x^2+4)2x}{(x^2+4)^4}\right]$

$=5\left[\frac{(x^2+4)(-2x)-(4-x^2)4x}{(x^2+4)^3}\right] = 5\left[\frac{2x^3-24x}{(x^2+4)^3}\right]$

$= \frac{10x(x^2-12)}{(x^2+4)^3}$ $\quad y''=0 \Rightarrow x=\pm 2\sqrt{3}$

$(-2\sqrt{3},0)\cup(2\sqrt{3},\infty)$ $\boxed{D}$

8. $3x^2=12 \Rightarrow x^2=4 \Rightarrow x=\pm 2$
at $x=-2, y=-5$ $\quad y+5=12(x+2)$ $\boxed{B}$

9. $y'=3x^2-3$ on $[-2,2]$
$y''=6x$ on $[-2,2]$
$f'(\pm 2)=9 \quad f'(0)=-3$ $\boxed{E}$

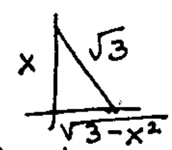
10. $2x-x\frac{dy}{dx}-y+2y\frac{dy}{dx}=0$ $\boxed{C}$
$\frac{dy}{dx}=\frac{y-2x}{2y-x}=\frac{4}{5} \quad \frac{4}{5}+\frac{-5}{4}=\frac{-9}{20}$

11. $V=s^3$ $\boxed{D}$
$dV=3s^2ds$
$dV=3(x_1)^2dx$

12. $\boxed{B}$

13. $\frac{f(4)-f(-4)}{4-(-4)} = c\cos c+\sin c$
$\frac{4\sin 4+4\sin(-4)}{8} = c\cos c+\sin c$
$0 = c\cos c+\sin c$
$c \doteq \pm 2.029, 0$ (calculator) $\boxed{A}$

14. $v(t)=6t^2-26t+22$
$a(t)=12t-26$
$a(t)=0$ at $t=13/6$
$v(1)=2$; $|v(13/6)|=6\ 1/6$;
$v(3.5)=4.5$ $\quad$ 37/6; left $\boxed{E}$

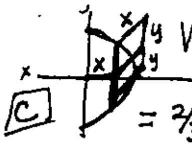
15.



$V(x)=\frac{1}{3}\pi(3-x^2)x$
$V'(x)=\pi-\pi x^2$
$\pi(1-x^2)=0$ at $x=1$
$h=x=1$; $r=\sqrt{3-1}=\sqrt{2}$
$h+r=1+\sqrt{2}$ $\boxed{B}$

16. $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$ $\boxed{D}$
$x_2=3-\frac{\tan 3}{\sec^2 3} \doteq 3.140$

17. $A(r)=8r^2+2\pi rh$
$A(r)=8r^2+2\pi r\left(\frac{1000}{\pi r^2}\right)$
$A(r)=8r^2+\frac{2000}{r}$
$A'(r)=16r-\frac{2000}{r^2}$
$16r^3-2000=0$
$r^3=1000/8$
$r=10/2=5$
$h=\frac{1000}{25\pi}=\frac{40}{\pi}$
$\frac{40}{\pi}\cdot\frac{1}{5}=8/\pi$ $\boxed{C}$

18. $\frac{dy}{dt}=k\sqrt{y}$; $\frac{dy}{\sqrt{y}}=k\,dt$
$2y^{1/2}=kt+c$; $c=2$
$y=\left(\frac{kt}{2}+1\right)^2$ $\boxed{A}$

19.



$V=\int_0 x\cdot 2\sqrt{1\ldots}$
$=\frac{2}{3}u^{3/2}\Big|_0^{1\ldots}$
$\boxed{C}$

20. $A=\int_0^3[(y+1)-(y-1)^2]d\ldots$
$\int_0^3(-y^2+3y)dy=\left(-\frac{1}{3}y^3+\frac{3}{2}\ldots\right.$
$9/2$ $\boxed{E}$

21. $\boxed{D}$ F and G are antiderivatives of ...

22. $\int\frac{\sin(4x-1)}{\cos^2(4x-1)}dx = \ldots$
$u=\cos(4x-1)$ $\quad =\frac{1}{4}u^{-1}$
$du=-4\sin(4x-1)$ $\quad \frac{1}{4}\sec(\ldots$ $\boxed{A}$

23. $\int_0^{\pi/4}\sqrt{1+\tan^2 x}\,dx=\int_0^{\pi/4}\sec x\,dx$
$\ln|\sec x+\tan x|\,\Big|_0^{\pi/4}=\ln(\sqrt{\ldots}$

24. $\frac{dy}{y(16-y)}=k\,dt$ or $\frac{1}{16}\int\ldots$
$\frac{1}{16}\ln y-\frac{1}{16}\ln(16-y)=kt+c$
$\frac{1}{16}(\ln 2-\ln 14)=c$; $c=\ldots$
$\frac{1}{16}\ln\left(\frac{y}{16-y}\right)=kt+\frac{1}{16}\ldots$
$\frac{1}{16}\ln\frac{4}{12}=50k+\frac{1}{16}\ln\ldots$
$\frac{1}{16}\ln\frac{7}{3}=50k$; $k=\ldots$

25. $\lim_{x\to 0}\frac{\frac{1}{x^2+1}-1}{24x^2}=\lim_{x\to 0}\ldots$
$-1/24$ $\boxed{E}$

26. $f'(x)=-\csc^2(x\sin\ldots$
$[x\cos x\ldots$
$=\frac{x\cos x+\sin x}{-\sin^2(x\sin x)}=\frac{x\cos\ldots}{\cos^2(\ldots}$
$\boxed{C}$

27. $\ln y=y\ln x$
$\frac{1}{y}\frac{dy}{dx}=\frac{y}{x}+\ln x\frac{dy}{dx}$
$\frac{dy}{dx}=\frac{\left(\frac{y}{x}\right)xy}{\left(\frac{1}{y}-\ln x\right)xy}=\frac{y^2}{x(1-\ln\ldots}$
$\frac{x^{2y}}{(1-\ln y)x}=\frac{x^{2y-1}}{1-\ln y}$

28. $=\lim_{x\to b}\frac{f(x)\cdot f(x)}{2}$

29. $\sum\frac{1}{2^n}=\frac{1}{1-\frac{1}{2}}$

30. $x-\frac{x^3}{3!}$ $\quad \frac{1}{2}-\ldots$
$\boxed{C}$ $\quad .4790\ldots$