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Armwood

Solutions

Calculus Individual Test March Regional

1. C $f(x)=\sqrt{4\sin x+2}$, $f'(x)=\dfrac{2\cos x}{\sqrt{4\sin x+2}}$, $f'(0)=\sqrt{2}$

2. C $y=\dfrac{x-1}{\sqrt{x}} \Rightarrow \dfrac{dy}{dx}=\dfrac{\sqrt{x}(1)-(x-1)\frac{1}{2}x^{-1/2}}{x}=\dfrac{x+1}{2x\sqrt{x}}$

3. A $y=x^2-4x+5 \Rightarrow \dfrac{dy}{dx}=2x-4$. The slope at x = 1 is -2. Equation is:
   (y - 2) = -2(x - 1) or 2x + y = 4.

4. B $y=\dfrac{1}{1-x^2} \Rightarrow \dfrac{dy}{dx}=\dfrac{2x}{(1-x^2)^2}$ The denominator is always positive , so dy/dx > 0 if 2x > 0, or x > 0.

5. D $x=f(t^2), y=g(t) \Rightarrow \dfrac{dx}{dt}=\dfrac{d(f(t^2))}{dt^2}\bullet\dfrac{dt^2}{dt}=f'(t^2)(2t)$
   $$\frac{dy}{dt}=\frac{d(g(t))}{dt}\bullet\frac{dt}{dt}$$
   $$\frac{dy}{dx}=\frac{dy}{dt}\bullet\frac{dt}{dx}\Rightarrow\frac{g'(t)}{2tf'(t^2)}$$

6. A $y=\dfrac{2}{4-x}\Rightarrow y'=\dfrac{2}{(4-x)^2}\Rightarrow y''=\dfrac{4}{(4-x)^3}$ We need y" < 0, which occurs at x > 4.

7. D We need to minimize the distance from a point P on xy = 4 to the origin, where P = (x,y). Dist. = $\sqrt{(x-0)^2+(\frac{4}{x}-0)^2} \Rightarrow D^2=\dfrac{x^4+16}{x^2}$
   Taking the derivative, we get $\dfrac{2x^4-32}{x^3}$ and the min. occurs at x = 2.
   Hence, P = (2, 2) and D = $2\sqrt{2}$.

8. B If f(x) = px^2 + qx + r, then $\dfrac{f(b)-f(a)}{b-a}=\dfrac{pb^2+qb+r-pa^2-qa-r}{b-a}$
   $$=p(b+a)+q.$$
   $f'(x)=2px+q$. Let $2px+q=p(b+a)+q$. And, $x=\frac{1}{2}(b+a)$.

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9. A Remember, that $e^{-x^2} = \frac{1}{e^{x^2}}$ and is always positive. As x $\to \infty$, $\frac{1}{e^{x^2}}$ will approach zero. Since cos x oscillates between 1 and -1, we 0 times a constant, which = 0.

10. C $y = 4\sin^2 x - \cos^2 x = 4(1-\cos^2 x) - \cos^2 x = 4 - 5\cos^2 x$. The maximum occurs where $5\cos^2 x = 0$, since $-5\cos^2 x \le 0$. Hence, y = 4.

11. B Method 1: Note that the function is the derivative of cos x at x = $\frac{\pi}{3}$. Or,

Method 2: By L'Hopital's Rule, we get $\lim_{x\to\frac{\pi}{3}} \frac{-\sin x}{1} = \frac{-\sqrt{3}}{2}$

12. C $s = 8t - 3t^2$. Average value = $\frac{1}{2+1}\int_{1}^{2}(8t - 3t^2)dt = \frac{5}{3}$.

13. C $x(t) = \frac{1-t}{1+t}$ and $v(t) = \frac{-2}{(1+t)^2}$ and $a(t) = \frac{4}{(1+t)^3}$. At t = 0, $a(t) = 4$.

14. D $\frac{dx}{dt} = \frac{1}{2}(3t^2+9)^{-1/2}(6t) = \frac{3t}{(3t^2+9)^{1/2}}$ and $\frac{dy}{dt} = 6t$.
$\frac{dy}{dx} = \frac{dy}{dt} \bullet \frac{dy}{dx} = (6t)\left(\frac{\sqrt{3t^2+9}}{3t}\right)$. At $t = \frac{-4}{\sqrt{3}}$, $\frac{dy}{dx} = 10$.

15. E To find $f'(2)$, we look at $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$ We need to look at the limit from both directions. From the right, we get 1 and from the left, we get -1. Since the limits are different, the limit does not exist.

16. B Area = $\int_0^4 (4-x)^{1/2}dx = \frac{16}{3}$.

17. A Length = $\int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \int_0^3 \sqrt{1+x}\,dx = \frac{14}{3}$.

18. D Over the interval of integration, the integral is positive, so we can just integrate 4 - x^2 from 2 to 4, and we get 32/3.

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19. E Remember, $e^{\ln x} = x$, so integrating xdx, we get 6 (not listed).

20. D (A) is true b/c if a function is differentiable for all x, then it is continuous for all x.
(B) is true b/c if a rel. max. occurs at (-4,32) and a rel. min. occurs at (0,0), then the function must be decreasing on [-4,0].
(C) is true b/c at (-4,32) and (0,0), $f'(0) = 0$.
(D) is <u>not necessarily</u> true b/c the Mean Value Thm. guarantees a point, c, in (-4,0) such that $f'(c) = \frac{f(0) - f(-4)}{0+4} = -8$, not $f'(c) = 8$.

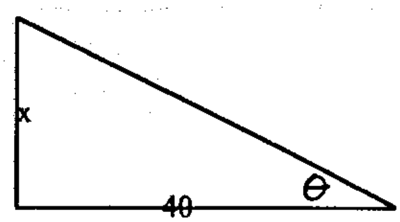
21. B The graph is a parabola with x-intercepts at 0 and 2k. Also, the area in question is below the x-axis. Hence, $A = -\int_0^{2k}(x^2 - 2kx)dx = 36$ and k = 3.

22. D If $f(x) = x^{2/3} - 1$, then $f'(x) = \frac{2}{3x^{1/3}}$. Thus, $f'(0)$ is not defined. And, f(x) is not differentiable on (-1,1). Therefore, Rolle's Thm. does not apply.

23. B Clearly, when t changes from 0 to 2π, P must make one revolution on a unit circle. Thus, the distance traveled is $2\pi r$, where $r = 1$. So, $d = 2\pi$.

24. D $V = \int_1^4 \pi(x^{-1})^2 dx = \frac{3\pi}{4}$

25. B $\frac{x}{40} = \tan\theta \Rightarrow x = 40\tan\theta$. $\frac{dx}{d\theta} = 40\sec^2\theta$ and $\frac{d\theta}{dt} = \frac{2\pi}{72}$. ($5° = \frac{1}{72}$ of 360°)
$\frac{dx}{dt} = \frac{dx}{d\theta} \bullet \frac{d\theta}{dt} = 40\sec^2(\frac{\pi}{6})(\frac{2\pi}{72}) = \frac{40\pi}{27}$.



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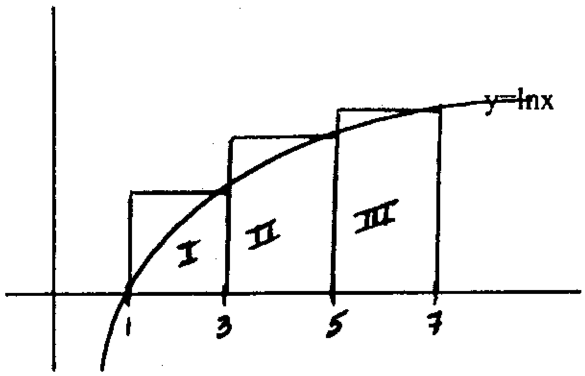
26. C



The base of each rect. is (7 -1)/3 = 2

Area of rectangle I = 2 f(3) = 2ln3
Area of rectangle II = 2f(5) = 2ln5
Area of rectangle III = 2f(7) = 2ln7

The total areas of all the rectangles is 2(ln3 + ln5 + ln7).

27. C $\lim_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right) = \lim_{h\to 0}\frac{1}{2}\bullet\frac{1}{\frac{1}{2}h}\ln(1+\frac{1}{2}h) = \lim_{\frac{1}{2}h\to 0}\frac{1}{2}\ln(1+\frac{1}{2}h)^{\frac{1}{\frac{1}{2}h}}$

$= \frac{1}{2}\ln\left(\lim_{\frac{1}{2}h\to 0}(1+\frac{1}{2}h)^{\frac{1}{\frac{1}{2}h}}\right) = \frac{1}{2}\ln e = \frac{1}{2}.$

28. D $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x)) \bullet g'(x)$, so $h'(1) = f'(2)g'(1) = 12$.

29. A y = ln2 - lnx and y ′= -1/x. y ′(2)= -1/2 (slope of tangent line). The slope of the normal line is 2.

30. A Let 1 + x = v, then g(v) = $v^n \ln v$. So, $\frac{d(f(x))}{d(1+x)} = \frac{d(g(v))}{dv} = v^n\frac{1}{v} + (\ln v)nv^{n-1}$
$= v^{n-1}(1+\ln v^n)$ or $(1+x)^{n-1}(1+n\ln(1+x))$.

ERRATA

#18 Calculus Individual

Corrected Solution

On solution, should say on the interval $[2,4]$

$\sqrt{(4-x^2)^2} = x^2-4$ so

integrate $\int_2^4 (x^2-4)\,dx$

answer D is Correct

#6 Calculus Team

The correct answer is 0. When you take the derivative with respect to x, t is a constant — the derivative of a constant is 0. For the answer to be $-2\sqrt{t^2+1}$, the question should have been $\frac{d}{dt}(\ldots)$.

ALG.2 14 C (all Es changed to C)

18 A (all Es changed to A)