Armwood

FAMAT REGIONAL
Calculus Individual
March 5, 1994

If none of the answers given is correct, choose e) NOTA.

1. $\int_0^2 e^{2x+3}\,dx =$

a) e^2-1 b) $\frac{e^2-1}{2}$ c) $\frac{e^3(e^4-1)}{2}$ d) $2e^7-2e^3$ e) NOTA

2. For what values of x is $y = 2^{x^2-2x+1}$ increasing ?

a) $x \geq 0$ b) $-1 < x < 1$ c) $x < -1$ or $x > 1$ d) $x > 1$ e) NOTA

3. $f(x) = 3 - x$ and $g(x) = 4x^2 - 2$. If $h(x) = g(f(x))$, find $h'(1)$.

a) -16 b) -8 c) 8 d) 16 e) NOTA

4. Let $F(x)=\int_0^{2x} \sqrt{\sec(t)}\,dt$. Approximate $F'(.5)$ to the nearest hundredt

a) 1.47 b) 1.36 c) 2.72 d) 3.72 e) NOTA

5. How many of the following graphs are symmetric about the origin ?
i) $y = \sec(x)\tan(x)$ ii) $y = |x|^3$ iii) $y = 3x^3 + 4x$ iv) $xy = 3$

a) one b) two c) three d) four e) NOTA

6. $\int_0^{\pi/2} |\cos(x) - \sin(x)|\,dx =$

a) 0 b) $\sqrt{2} - 1$ c) $2\sqrt{2}$ d) $2\sqrt{2} - 2$ e) NOTA

7. $y = \text{arcsec}(3x)$. Find the slope of the normal to the graph at x=1.

a) $-6\sqrt{2}$ b) $\frac{-3\sqrt{2}}{4}$ c) $\frac{\sqrt{2}}{4}$ d) $2\sqrt{2}$ e) NOTA

8. Find the average value of $f(x) = \sin(2x-\pi/4)$ from $x = 0$ to $x = \pi/2$

a) 0 b) $\frac{\sqrt{2}}{\pi}$ c) $\frac{2\sqrt{2}}{\pi}$ d) $\frac{\pi\sqrt{2}}{2}$ e) NOTA

9. Find the area of the region bounded by the graphs of $y + x^2 = 6$ and $y + 2x - 3 = 0$.

a) 15/4 b) 32/3 c) 46/3 d) 16 e) NOTA

10. The velocity at time t, $t \geq 0$ of a point moving on a straight line is $v(t) = (1 + t^2)^{-1}$ ft/sec. If the point is at the origin at t = 0, find its position at the instant that the acceleration and velocity have the same absolute value.

a) $\pi/4$ b) 1/2 c) $\pi/3$ d) 1 e) NOTA

11. $\int_0^4 (x^2 - 4)\, dx =$

a) 8/3 b) 16/3 c) 16 d) 48 e) NOTA

12. The region bounded by $y = 2x^2$, $y = 0$, and $x = 2$ is rotated about the y-axis. What is the volume of the resulting solid ?

a) $16\pi/3$ b) $128\pi/5$ c) $64\pi/3$ d) 8π e) NOTA

13. $\lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x - 3} =$

a) 1/4 b) 1/2 c) 1 d) does not exist e) NOTA

14. At what point(s) does the graph of $f(x) = x^2(x-1)^{-1}$ have a horizontal tangent ?

a) (0,0),(-2,-4/3) b)(0,0),(4,2)
c)(-2,-4/3),(2,4) d)(0,0),(2,4) e) NOTA

15. $f(x) = \sqrt[3]{\sin^2(2x)}$. $f'(x) =$

a) $\dfrac{2}{3\sqrt[3]{\sin(2x)}}$ b) $\dfrac{4}{3\sqrt[3]{\sin(2x)}}$ c) $\dfrac{4\cos(2x)}{3\sqrt[3]{\sin(2x)}}$ d) $\dfrac{4\sqrt[3]{\cot(2x)}}{3}$ e) NOT

16. Find the equation of the curve passing through (0,4/3) given that $y' = x\sqrt{1 - x^2}$.

a) $y = \dfrac{-\sqrt{(1-x^2)^3}}{3} + \dfrac{5}{3}$ b) $y = \dfrac{-1}{4\sqrt{1-x^2}} + \dfrac{7}{12}$

c) $y = \dfrac{2\sqrt{(1-x^2)^3}}{3} + \dfrac{2}{3}$ d) $y = \dfrac{\sqrt{(1-x^2)^3}}{3} + 1$ e) NOTA

17. $\int 2\sec^2(x)(1-\cos^2(x))\,dx =$

a) $2\tan(x) - x + C$ b) $\frac{2\tan^3(x)}{3} + C$

c) $2x - \frac{2\cos^3(x)}{3} + C$ d) $2\tan(x) - 2x + C$ e) NOTA

18. $h(x) = f(x)\cdot g(x)$. f and g are always positive and have first and second derivatives for all x in their domain. Which of the following must be true ?

a) If f and g both have a relative max at x=a, then so does h.
b) If f and g both have a point of inflection at x=a, then so does h.
c) h' is always negative. d) h is always increasing. e) NOT

19. $\int \frac{\cos(x)}{\sqrt{4 - \sin^2(x)}}\,dx =$

a) $2\sqrt{4 - \sin^2(x)} + C$ b) $\arcsin(\sin(x)/2) + C$
c) $2\arcsin(\sin(x)/2) + C$ d) $\sin(\arccos(x/2) + C$ e) NOTA

20. Air is pumped into a spherical balloon at the rate of 4.5 in^3/min. To the nearest hundredth find the rate of change of the radius when the radius is 2.

a) .045 b) .09 c) 113.04 d) 226.08 e) NOTA

21. $\lim_{x\to\infty}\left(\sqrt{a^2x^2 + bx + 1} - \sqrt{a^2x^2 + 1}\right) =$

a) 0 b) $b/(2|a|)$ c) $b/(|a|)$ d) $b/2$ e) NOTA

22. Where is the graph of $f(x) = x^4 - 6x^2 + 1$ concave down ?

a) $x < -\sqrt{3}$ or $0 < x < \sqrt{3}$ b) $1 < x < \sqrt{3}$ or $-\sqrt{3} < x < -$
c) $x < -1$ or $x > 1$ d) $-1 < x < 1$ e) NOTA

23. The area bounded by $y = \sqrt{\sec(x)}$, $x = 0$, $y = 0$, and $x = \pi/4$ is rotated about the x-axis. Find the volume of the resulting solid.

a) π b) $\pi(\sqrt{2}+1)$ c) $\ln(\sqrt{2}+1)$ d) $\pi[\ln(\sqrt{2}+1)]$ e) NOT

24. $\int \frac{\sqrt{9 - x^2}}{x}\, dx =$

a) $3\ln\left|\frac{3-\sqrt{9-x^2}}{x}\right| + \sqrt{9-x^2} + C$

b) $\frac{\sqrt{(9-x^2)^3}}{27} + C$

c) $3\ln\left|\frac{\sqrt{(9-x^2)^3}}{27}\right| + C$

d) $\ln\left|\frac{x}{3}\right| + C$

25. Write the equation of the tangent to the graph of xy=8 at x=2.

a) 2x-y=0 b) x+2y=6 c) 2x+y=8 d) x-2y=-6 e)NOTA

26. Using the trapezoidal rule and n = 4, estimate the area under the curve of $y = x^2$ from x = 0 to x = 2.

a) 3/2 b) 8/3 c) 11/2 d) 11/4 e) NOTA

27. $\frac{dy}{dx} = \frac{2}{e^y}$. If y = 0 when x = 5, find y as a function of x.

a) $e^y=2x+9$ b) $y=2\ln|x-4|$ c) $y=\ln|2x-9|$ c) $y=\ln|x-9|$ e) NOTA

28. At what time t, $t \geq 0$, will the instantaneous velocities of the following functions be equal. $x(t)=\arctan(t)$ $s(t)=\ln|t+13|$

a) t = 3 b) t = 4 c) $t = \frac{-1+\sqrt{55}}{2}$ d) $t = \frac{-1+\sqrt{57}}{2}$ e) NOTA

29. If y" = (-1/4)y, which of the following could be y ?

a) $e^{(-1/2)x}$ b) cos(2x) c) (1/4)sin(x) d) sin(x/2) e) NOT

30. $\int_1^4 \ln(2x+1)\, dx =$

a) $\frac{15\ln3 - 6}{2}$

b) $\frac{9\ln9 - 3\ln3 - 12}{2}$

c) $3\ln3 - 6$

d) $9\ln9 - 3\ln3 - 6$

e) NOTA