3

Arnwood

FAMAT REGIONAL CALCULUS MARCH 5, 1994

C 1. $u = 2x+3$, $du = 2\,dx$ $\quad \frac{1}{2}\int_3^7 e^u\,du = \frac{1}{2}(e^7 - e^3) = \frac{e^3(e^4-1)}{2}$

D 2. $y' = 2^{x^2-2x+1}\cdot \ln 2 \cdot 2x - 2$ $\quad 2(x-1)$ inc when $x > 1$.

A 3. $g(s(x)) = 4(3-x)^2 - 2$ $\quad h'(x) = 8(3-x)(-1) = 8(2)(-1) = -16$

C 4. $F'(x) = 2\sqrt{\sec(2x)}$ $\quad F'(.5) = 2\sqrt{\sec 1} = 2.72$

C 5. 3 i, iii, iv

D 6. $2\int_0^{\pi/4}(\cos x - \sin x)\,dx = 2\Big|_0^{\pi/4} \sin x + \cos x = 2\left[\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right] = 2\sqrt{2} - 2$

E 7. $y' = \frac{3}{|3x|\sqrt{9x^2-1}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$ $\quad$ m norm $= -2\sqrt{2}$

B 8. $\frac{1}{\pi/2 - 0}\int_0^{\pi/2}\sin(2x - \pi/4)\,dx$ $\quad u = 2x - \pi/4$, $du = 2\,dx$ $\quad \frac{1}{\pi}\cdot\frac{2}{2}\int_{-\pi/4}^{3\pi/4}\sin u\,du = -\frac{1}{\pi}\left(\cos\frac{3\pi}{4} - \cos\left(-\frac{\pi}{4}\right)\right) = \frac{\sqrt{2}}{\pi}$

B 9. $6 - x^2 = 3 - 2x$ $\quad (x-3)(x+1)$ $\quad \int_{-1}^{3}[(6-x^2)-(3-2x)]\,dx$ $\quad \int_{-1}^{3}(-x^2+2x+3)\,dx = \Big|_{-1}^{3} -\frac{x^3}{3} + x^2 + 3x = (-9+9+9) - (\frac{1}{3}+1-3) = \frac{32}{3}$

A 10. $x(t) = \arctan t$ $\quad a(t) = -1(1+t^2)^{-2}\cdot 2t = \frac{-2t}{(1+t^2)^2}$ $\quad \frac{1}{1+t^2} = \left|\frac{-2t}{(1+t^2)}\right|$ $\quad$ 1ST $1 = \frac{2t}{1+t^2}$ $\quad t^2+1 = 2t$ $\quad t^2 - 2t + 1 = 0$ $\quad (t-1)^2 = 0$ $\quad \boxed{t=1}$ $\quad$ 2ND $1 = \frac{-2t}{1+t^2}$ $\quad t^2+1 = -2t$ $\quad t^2+2t+1$ $\quad (t+1)^2 = 0$ $\quad \boxed{t=-1}$

B 11. $\arctan 1 = \pi/4$ $\quad \int_0^4 (x^2-4)\,dx = \Big|_0^4 \frac{x^3}{3} - 4x = \frac{64}{3} - 16 = \frac{64}{3} - \frac{48}{3} = \frac{16}{3}$

E 12. $2\pi\int_0^2 x(2x^2)\,dx = 2\pi\int_0^2 2x^3\,dx = 2\pi\Big|_0^2 \frac{1}{2}x^4 = \pi(16-0) = 16\pi$

A 13. $\lim_{x\to 3}\frac{1}{2}(x+1)^{-1/2} = \frac{1}{4}$

D 14. $5'(x) = \frac{(x-1)(2x) - x^2}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$ $\quad x = 0,\ y = 0$ $\quad x = 2,\ y = 4$

C 15. $\left[\sin(2x)\right]^{2/3}$ $\quad 5'(x) = \frac{2}{3}\left[\sin(2x)\right]^{-1/3}\cdot 2\cos 2x = \frac{4\cos 2x}{3\sqrt[3]{\sin(2x)}}$ $\quad$ at $x = 0$

A 16. $\int x\sqrt{1-x^2}\,dx$ $\quad u = 1-x^2$, $du = -2x\,dx$ $\quad -\frac{1}{2}\int u^{1/2}\,du = -\frac{1}{3}u^{3/2} + C$ $\quad \frac{y}{3} = -\frac{\sqrt{(1-x^2)^3}}{3} + C$ $\quad \frac{4}{3} = -\frac{1}{3} + C$ $\quad C = 5/3$ $\quad = -\frac{\sqrt{(1-x^2)^3}}{3} + \frac{5}{3}$

D 17. $\int(2\sec^2 x - 2)\,dx = \boxed{2\tan x - 2x + C}$

A 18. $h'(x) = s(x)\cdot g'(x) + g(x)\cdot s'(x)$ $\quad$ If s and g both have a max at a then $\therefore h'(x)$

If $s'(a) = 0$ and $g'(a) = 0$, then $h'(a) = 0$

	$s'(x)$		$g'(x)$		$h'(x)$
$(-\infty, a)$	+	$(-\infty, a)$	+	$(-\infty, a)$	+
(a, ∞)	−	(a, ∞)	−	(a, ∞)	−

b, c, and d are false.

B 19. $a = 2$ $\quad u = \sin x$ $\quad du = \cos x\,dx$ $\quad \int\frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C = \boxed{\arcsin\left(\frac{\sin x}{2}\right) + C}$

B 20. $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$ $\quad 4.5 = 4\pi(4)\left(\frac{dr}{dt}\right)$ $\quad \frac{dr}{dt} = \frac{4.5}{16\pi} = \boxed{.09}$

B 21. $\lim_{N\to\infty}\sqrt{a^2x^2+bx+1} - \sqrt{a^2x^2+1}$ $\quad \frac{\sqrt{a^2x^2+bx+1}+\sqrt{a^2x^2+1}}{\sqrt{a^2x^2+bx+1}+\sqrt{a^2x^2+1}} = \lim_{N\to\infty}\frac{bx}{\sqrt{a^2x^2+bx+1}+\sqrt{a^2x^2+1}} = \lim_{N\to\infty}\frac{bx}{2|a|x}$ $\quad \boxed{\frac{b}{2|a|}}$

$\boxed{-1 < x < 1}$

D 22. $s'(x) = 4x^3 - 12x$ $\quad 12(x^2-1)$ $\quad s''(x) = 12x^2 - 12$ $\quad 12(x+1)(x-1)$

D 23. $\pi\int_0^{\pi/4}\sec x\,dx = \pi\Big|_0^{\pi/4}\ln|\sec x + \tan x| = \pi\left[\ln|\sqrt{2}+1| - \ln 1\right] = \boxed{\left[\ln(\sqrt{2}+1)\right]\pi}$

A 24. $x = 3\sin\theta$ $\quad dx = 3\cos\theta\,d\theta$ $\quad \int\frac{3\cos\theta\cdot 3\cos\theta\,d\theta}{3\sin\theta} = 3\int\frac{\cos^2\theta\,d\theta}{\sin\theta\,d\theta} = 3\int\frac{1-\sin^2\theta}{\sin\theta}\,d\theta = 3\int(\csc\theta - \sin\theta)\,d\theta$ $\quad 3\left[\ln|\csc\theta - \cot\theta| + \cos\theta\right] + C$

$\int\frac{3}{\sqrt{9-x^2}}\,dx$ $\quad \left|3\ln\left|\frac{3}{x} - \frac{\sqrt{9-x^2}}{x}\right| + \frac{3\sqrt{9-x^2}}{3} + C\right.$

C 25. $y = 8x^{-1}$ $\quad y' = -8x^{-2}$ $\quad m_{tan} = -2$ $\quad x = 2,\ y = 4$ $\quad \boxed{2x + y = 8}$

D 26. $\frac{2}{8}\left[0 + 2(\frac{1}{2})^2 + 2(1) + 2(\frac{3}{2})^2 + (2)^2\right] = \frac{1}{4}\left[11\right] = \frac{11}{4}$

C 27. $\int e^y\,dy = \int 2x\,dx$ $\quad e^y = 2x + C$ $\quad 1 = 18 + C$ $\quad C = -9$ $\quad e^y = 2x - 9$ $\quad y = \ln|2x - 9|$

B 28. $\frac{1}{1+x^2} = \frac{1}{x+13}$ $\quad x + 13 = 1 + x^2$ $\quad 0 = x^2 - x - 12$ $\quad (x-4)(x+3)$ $\quad t = 4$

D 29. $\sin(\pi/2)$ $\quad \frac{1}{2}\cos(\pi/2)$ $\quad -\frac{1}{4}\sin(\pi/2)$

A 30. $u = 2x+1$ $\quad du = 2\,dx$ $\quad \frac{1}{2}\int_3^9 \ln u = \frac{1}{2}\Big|_3^9 u\ln u - u = \frac{1}{2}\left[(9\ln 9 - 9) - (3\ln 3 - 3)\right] = \left[18\ln 3 - 9 - 3\ln 3 + 3\right]\cdot\frac{1}{2}$ $\quad \frac{15\ln 3 - 6}{2}$